4.1.2
\[ v = -14.97 \text{ m/s} \]

4.1.8
\[ \mu = 0.035 \]

4.1.21
\[ \bar{v}_m = -1.73 \bar{v}_\theta \text{ m/s} \]

4.2.28
\[ L = 9.67 \text{ m} \]

\[ v = 7.95 \text{ m/s} \]

4.3.12
\[ v_2 = 8.34 \text{ mph} \]

4.3.21
\[ \eta = 0.872 \text{ and we have an efficiency of } 87.2 \text{ percent}. \]
Bonus Problems

5.1.16

\[ \hat{a}_A = \hat{a}_B = 0.746 \text{ m/s}^2 \]

5.1.23

False.
Recommended Problems

4.1.3

GOAL: Find speed of arrow after it has moved 1.6 feet
GIVEN: arrow’s weight and force profile
DRAW:

\[ \begin{array}{c}
\text{f} \\
\downarrow \\
\bullet \\
\downarrow \\
x
\end{array} \]

FORMULATE EQUATIONS:

\[ \kappa \mathcal{E} \bigg|_2 = \kappa \mathcal{E} \bigg|_1 + W_{1-2} \]

SOLVE:

\[ m = \frac{(20 \text{ oz})}{(16 \text{ oz/lb})(32.2 \text{ ft/s}^2)} = 3.88 \times 10^{-3} \text{ slug} \]

\[ \mathcal{E} \bigg|_1 = 0 \]

\[ W_{1-2} = \int_0^{1.6} 40e^{-3.2x} dx \text{ ft-lb} \]

\[ = \left[ -\frac{40}{3.2} e^{-3.2x} \right]_0^{1.6} \text{ ft-lb} = 12.4 \text{ ft-lb} \]

\[ \frac{1}{2} m \dot{x}^2 = W_{1-2} \]

\[ \frac{1}{2} (3.88 \times 10^{-3} \text{ slug}) \dot{x}^2 = 12.4 \text{ ft-lb} \]

\[ \dot{x} = 80 \text{ ft/s} \]
4.1.9

GOAL: Determine the coefficient dynamic friction needed to bring a sliding mass to rest after rising 5 m.

GIVEN: Mass, slope and initial speed.

DRAW:

![Diagram showing mass sliding on a slope with forces and vectors labeled.]

FORMULATE EQUATIONS: The FBD gives the forces on the mass as

\[ N\vec{b}_2 + \mu N\vec{b}_1 - mg\vec{f} \]

In addition to this, we'll use our work/energy formulation:

\[ \frac{1}{2}mv_1^2 + W_{1-2} = \frac{1}{2}mv_2^2 \]

SOLVE: The forces we need be concerned with (the ones that act along the path) are given by

\[ F = mg\sin \theta - \mu N \]

A force balance in the \( \vec{b}_2 \) direction gives us \( N = mg/\sqrt{2} \) and so, using \( \theta = 45^\circ \), our downslope force becomes

\[ F = \frac{mg(1 - \mu)}{\sqrt{2}} \]

First let's consider the case of \( \mu = 0 \). In this case we have

\[ 0 + \frac{mgd}{\sqrt{2}} = \frac{1}{2}mv_2^2 \]

\[ \frac{(9.81 \text{ m/s}^2)(10 \text{ m})}{\sqrt{2}} = \frac{v_2^2}{2} \Rightarrow v_2 = 11.8 \text{ m/s} \]

Next, we'll let \( \mu = 0.1 \):

\[ 0 + \frac{mgd(1 - \mu)}{\sqrt{2}} = \frac{1}{2}mv_2^2 \]

\[ \frac{(9.81 \text{ m/s}^2)(10 \text{ m})(1.0 - 0.1)}{\sqrt{2}} = \frac{1}{2}v_2^2 \Rightarrow v_2 = 11.2 \text{ m/s} \]

The mass is moving 0.604 m/s slower due to the friction, a 5.1% decrease.
4.1.20

GOAL: Determine the tension in a pair of restraining strings and the height above the floor attained by the released block.

GIVEN: Spring characteristics and mass of block.

DRAW:

FORMULATE EQUATIONS:
To solve the problem we'll use the energy/work formula:
\[
\frac{1}{2}mv_1^2 + W_{1-2} = \frac{1}{2}mv_2^2
\]

SOLVE:
We initially have a static force balance in the vertical direction:
\[
-2T \cos 30^\circ - mg + 2k\Delta x = 0
\]
\[
T = \frac{2(500 \text{ N/m})(0.05 \text{ m}) - mg}{2 \cos 30^\circ} = \frac{50 \text{ N} - mg}{\sqrt{3}}
\]

We see from this that as the weight of the block (mg) increases, the tension decreases and, for a sufficiently heavy block, the strings would go slack.

Now we move onto the case when the strings break. We'll consider three total states. State 1 is with the block at its lowest point (0.01 m above the floor with the spring compressed 0.05 m). State 2 is when the spring has reached its fully extended state (block is 0.06 m above the floor with a speed to be determined). After state 2 the block is in a free trajectory and will reach a maximum height when gravity has decreased its speed to zero (state 3).

State 1 to state 2:
\[
\frac{2k(\Delta x)^2}{2} - mg(0.05 \text{ m}) = \frac{1}{2}mv_2^2
\]

From state 2 to state 3 we have
\[
\frac{1}{2}mv_2^2 = mg\Delta y
\]

(1), (2) \Rightarrow
\[
\frac{2k(\Delta x)^2}{2} - mg(0.05 \text{ m}) = mg\Delta y
\]
\[
\Delta y = -0.05 \text{ m} + \frac{k(\Delta x)^2}{2mg}
\]

The total height \( h \) is the height at state 2 (0.06 m) plus the change in height from state 2 to state 3 (\( \Delta y \)). Thus we have
4.2.6

4.2.6

GOAL: Find spring constant to limit spring compression to specified amount.
GIVEN: Initial speed and parameters. \( L = 5 \text{ m}, m = 2 \text{ kg}, \theta = 30^\circ, v = 8 \text{ m/s}. \)
DRAW:

\[ h = 0.06 \text{ m} + \Delta y = 0.01 \text{ m} + \frac{2k(\Delta x)^2}{2mg} = 0.01 \text{ m} + \frac{(500 \text{ N})(0.05 \text{ m})^2}{2m(9.81 \text{ m/s}^2)} \]

\[ h = 0.01 \text{ m} + \frac{0.127 \text{ kg} \cdot \text{ m}}{m} \]

Clearly the mass of the block will alter the ultimate height attained (as expected from physical considerations). Note that the analysis assumes that the mass isn’t so great as to prevent the spring from completely extending. The way to determine if this assumption is valid is to simply evaluate \( h \) for a given value of \( m \). If \( h \) exceeds 0.06 m, the unstretched spring length, then we know that \( m \) was “small enough” to match our assumption.

FORMULATE EQUATIONS: We’ll apply conservation of energy:

\[ \kappa \mathcal{E}_2 + \mathcal{P}_2 = \kappa \mathcal{E}_1 + \mathcal{P}_1 \]

SOLVE:

(1) \[ 0 + \frac{1}{2} k(0.1 \text{ m})^2 + mg(L + 0.1 \text{ m}) \sin \theta = \frac{1}{2} mv^2 \]

(2) \[ k = \frac{(2 \text{ kg})((8 \text{ m/s})^2 - (9.81 \text{ m/s}^2)(5.1 \text{ m}))}{0.1 \text{ m}^2} = 2.79 \times 10^3 \text{ N/m} \]
4.2.29

**GOAL:** Find the maximum deflection of the spring after a moving mass strikes it.

**GIVEN:** System parameter values.

**DRAW:**

\[
\begin{pmatrix}
\vec{b}_1 \\
\vec{b}_2
\end{pmatrix} \begin{bmatrix}
\sin \beta \\
-\cos \beta \\
\cos \beta \\
\sin \beta
\end{bmatrix} = m \ddot{x}
\]


**FORMULATE EQUATIONS:** A force balance gives us

\[N \vec{b}_2 - S \vec{b}_1 - mg \vec{f} = m \ddot{x} \vec{b}_1\]

If the mass slides then \(S = \mu N\). Resolving the equations of motion into the \(\vec{b}_1\) and \(\vec{b}_2\) directions yields:

\[
mg \cos \beta - \mu N = m \ddot{x} \\
N - mg \sin \beta = 0
\]

Which, when combined, give us:

\[m \ddot{x} = mg(\cos \beta - \mu \sin \beta)\]

Thus, the force acting on the mass along the direction of the travel is \(mg(\cos \beta - \mu \sin \beta)\).

Work-energy gives us

\[\delta E|_1 + \int \delta E_1|_1 + W_{1-2} = \delta E|_2 + \int \delta E_2|_2\]

**SOLVE:** At top of the slide

\[\delta E|_1 = 0, \quad \delta E_1|_1 = mgL \cos \beta\]

At the instant of contact with the spring we have

\[\delta E|_2 = \frac{1}{2}mv^2_0, \quad \delta E_2|_2 = -mgL \cos \beta, \quad W_{1-2} = -\mu NL\]

Invoking the work-energy between these two states

\[-\mu mgL \sin \beta = \frac{1}{2}mv^2_0 - mgL \cos \beta\]

\[-0.1(9.81 \text{ m/s}^2)(2 \text{ m})\frac{1}{2} = \frac{1}{2}v^2_0 - (9.81 \text{ m/s}^2)(2 \text{ m}) \sqrt{\frac{3}{2}}\]
\( v_2 = 5.66 \text{ m/s} \). At full compression we have

\[
\kappa\varepsilon_3|_3 = 0, \quad \mathcal{P}\varepsilon_3|_3 = 0, \quad \mathcal{P}\varepsilon_3|_3 = \frac{1}{2} k(\Delta x)^2
\]

Using work-energy between contact and full compression gives

\[
\frac{1}{2}mv_2^2 - \mu mg \sin \beta \Delta x = -mg \cos \beta \Delta x + \frac{1}{2}k(\Delta x)^2
\]

\[
\frac{1}{2}(1.1 \text{ kg})v_2^2 - 0.1(1.1 \text{ kg})(9.81 \text{ m/s}^2)\frac{1}{2}\Delta x = -(1.1 \text{ kg})(9.81 \text{ m/s}^2)\frac{\sqrt{3}}{2}\Delta x + \frac{1}{2}(3500 \text{ N/m})(\Delta x)^2
\]

\[
(\Delta x)^2 - (5.032 \times 10^{-3} \text{ m})(\Delta x) - 1.006 \times 10^{-2} \text{ m}^2 = 0
\]

\[\Delta x = 0.1029 \text{ m}\]

4.3.16

GOAL: Determine a motor’s efficiency.

GIVEN: Mass of load, speed of load, slope’s inclination and electrical power input.

DRAW:

![FBD Diagram]

FORMULATE EQUATIONS:

We’ll use our FBD to determine the tension applied to the load and then use \( P = Tv \) to find the power and \( \eta = P_{out}/P_{in} \) to find the efficiency.

SOLVE:

From the FBD we have

\[
T - mg \sin \theta = 0
\]

\[
T = mg \sin \theta = (1000 \text{ kg})(9.81 \text{ m/s}^2)(0.5) = 4905 \text{ N}
\]

\[
P = Tv = (4905 \text{ N})(2 \text{ m/s}) = 9810 \text{ N} \cdot \text{m/s} = 9810 \text{ W}
\]

\[
\eta = \frac{9810 \text{ W}}{18 \times 10^3 \text{ W}} = 0.545
\]

Thus we see that \( \eta = 0.545 \) and we have an efficiency of \( 54.5 \text{ percent} \).