Humming Bird problem [10 pts]

\[ \omega = \int f_s \, ds = \frac{\omega}{5} \]

where \( f_s = 2 (mg) = 2 \times 0.2 \, \text{kg} \times 9.81 \, \text{m/s}^2 = 3.924 \, \text{N} \)

\[ S = \frac{2 \pi r \theta}{360^\circ} = \frac{2 \pi \left( 5 \times 10^{-2} \, \text{m} \right)}{360^\circ} = 0.069 \, \text{m} \]

\[ \omega = \frac{f_s}{S} = 0.274 \, \text{J/stroke} \]

\[ W = \frac{\text{Work}}{S} = 0.274 \, \text{J/stroke} \times 100 \, \text{strokes} \times \frac{746}{746} = 0.04 \, \text{J} \]
4.1.2 \( (10 \text{ pts}) \)

**GOAL:** Find the speed of the mass when it is 0.6 m from the wall

**GIVEN:** \( m = 0.5 \text{ kg}, \ k = 40 \text{ N/m}, \) unstretched length \( L_0 = 0.3 \text{ m}, \ x_1 = 2 \text{ m}, \ x_2 = 0.6 \text{ m} \)

**DRAW:**

\[
\begin{align*}
\text{STATE 1} & \quad \frac{x}{J} \quad m \quad \square \quad L_0 \\
\text{STATE 2} & \quad \frac{\text{wall}}{m}
\end{align*}
\]

**ASSUME:** The surface is frictionless and the spring is linear.

**FORMULATE EQUATIONS:**

We'll apply work/energy:

\[
\begin{align*}
\mathcal{KE}_2 & = \mathcal{KE}_1 + W_{1-2} \\
\mathcal{KE}_1 & = 0 \implies \mathcal{KE}_2 = W_{1-2}
\end{align*}
\]

**SOLVE:**

\[
W_{1-2} = \int_{x_1}^{x_2} F_{\text{ext}} \, dt = \int_{x_1}^{x_2} -(k(x - L_0)) \, dx
\]

\[
W_{1-2} = -k \int_{x_1}^{x_2} (x - L_0) \, dx = -k \left[ \frac{x^2}{2} - L_0 x \right]_{x_1}^{x_2}
\]

\[
W_{1-2} = -(40 \text{ N/m}) \left[ \left( \frac{(0.6 \text{ m})^2}{2} - (0.3 \text{ m})(0.6 \text{ m}) \right) - \left( \frac{(2 \text{ m})^2}{2} - (0.3 \text{ m})(2 \text{ m}) \right) \right] = 56 \text{ J}
\]

\[
\mathcal{KE}_2 = W_{1-2} = 56 \text{ J} = \frac{1}{2} mv^2 \implies v = 14.97 \text{ m/s}
\]

\[
\dot{v} = -14.97 \text{ m/s}
\]
4.1.8 (to p/s)

GOAL: Determine the coefficient dynamic friction needed to bring a sliding mass to rest after rising 5 m.

GIVEN: Mass, slope and initial speed.

DRAW:

\[
\begin{bmatrix}
\hat{b}_1 & \hat{b}_2 \\
\cos \theta & -\sin \theta \\
\sin \theta & \cos \theta \\
\end{bmatrix}
\]

FORMULATE EQUATIONS: The FBD gives the forces on the mass as

\[
N\hat{b}_2 - \mu N\hat{b}_1 - mg\hat{f}
\]

In addition to this, we'll use our work/energy formulation:

\[
\frac{1}{2}mv_1^2 + W_{1-2} = \frac{1}{2}mv_2^2
\]

SOLVE: The forces we need be concerned with (the ones that act along the path) are given by

\[
F = -\mu N - mg \sin \theta
\]

A force balance in the \( \hat{b}_2 \) direction gives us \( N = mg \cos \theta \) and so our applied force becomes

\[
F = -mg(\mu \cos \theta + \sin \theta)
\]

Applying work/energy from state 1 to state 2 gives us

\[
\frac{1}{2}mv_1^2 - mg(\mu \cos \theta + \sin \theta)d = 0
\]

\[
\mu = \frac{v_1^2 - 2g \sin \theta d}{2g \cos \theta d} = \frac{(10.2 \text{ m/s})^2 - 2(9.81 \text{ m/s}^2)(0.5)(10 \text{ m})}{2(9.81 \text{ m/s}^2)(0.866)(10 \text{ m})}
\]

\[
\mu = 0.035
\]
4.1.21 (10 pts)

GOAL: Determine the speed of a mass particle at $|\theta| = 30^\circ$ along a circular path.

GIVEN: Particle's mass, shape of path, initial velocity and position.

DRAW:

FORMULATE EQUATIONS:
To solve the problem we'll use the energy/work formula:

$$\frac{1}{2}mv_1^2 + W_{1-2} = \frac{1}{2}mv_2^2$$

SOLVE:
We'll first determine if the mass has enough energy to reach the top of the hill with a finite speed. If so, we know that it will then move to the right side of the hill ($\theta$ negative) and thus will eventually reach $\theta = -30^\circ$. If it turns out not to have enough energy to reach $\theta = 0$ then the conclusion is that it stops somewhere partway up, reverses direction and eventually reaches $\theta = 30^\circ$. The force due to gravity that acts against the mass along its trajectory is $mg\sin\theta$.

From state 1 to state 2 we have

$$\frac{1}{2}mv_1^2 + \int_{\theta_0}^{0} mg\sin\theta r d\theta = \frac{1}{2}mv_2^2$$

$$v_2^2 = 2 \left[ \frac{1}{2} (1.25\text{ m/s})^2 - (9.81\text{ m/s}^2)(1\text{ m})(1 - \cos 20^\circ) \right]$$

$$v_2^2 = 0.379\text{ (m/s)}^2$$

$v_2^2$ is positive, implying a real solution. Our conclusion is that it does reach $\theta = 0$ and therefore will pass $\theta = -30^\circ$.

Denoting its position at $\theta = 30^\circ$ as state 3 we have

$$\frac{1}{2}mv_1^2 + \int_{\theta_0}^{-30^\circ} mg\sin\theta r d\theta = \frac{1}{2}mv_3^2$$

$$v_3^2 = 2 \left[ \frac{1}{2} (1.25\text{ m/s})^2 - (9.81\text{ m/s}^2)(1\text{ m})(\cos(-30^\circ) - \cos 20^\circ) \right]$$

$$v_3 = -1.73 \text{ m/s}$$
4.2.28 (Out of this section)

**GOAL:** Find safe length of bungie cord and impact velocity if cord is too weak.

**GIVEN:** Initial and final height of jumper, mass of jumper and spring constant of bungie cord.

**DRAW:**

- 70 m --- JUMP HEIGHT (STATE 1)
- 3 m --- SPEED = ZERO HEIGHT (STATE 2)
- 0 --- GROUND

**FORMULATE EQUATIONS:** Conservation of energy

\[ K\mathcal{E}|_1 + P\mathcal{E}|_1 = K\mathcal{E}|_2 + P\mathcal{E}|_2 \]  

(1)

**SOLVE:**

(a) Kinetic and potential energies

\[ K\mathcal{E}|_1 = 0, \quad P\mathcal{E}_{g}|_1 = mg(70 \text{ m}), \quad P\mathcal{E}_{bc}|_1 = 0 \]  

(2)

\[ K\mathcal{E}|_2 = 0, \quad P\mathcal{E}_{g}|_2 = mg(3 \text{ m}), \quad P\mathcal{E}_{bc}|_2 = \frac{1}{2}k(67 \text{ m} - L)^2 \]  

(3)

Using (1) we get

\[ (55 \text{ kg})(9.81 \text{ m/s}^2)(70 \text{ m}) = (55 \text{ kg})(9.81 \text{ m/s}^2)(3 \text{ m}) + \frac{1}{2}(22 \text{ N/m})(67 \text{ m} - L)^2 \]  

(4)

\[ L = 9.67 \text{ m} \]

(b) Kinetic and potential energies

\[ K\mathcal{E}|_1 = 0, \quad P\mathcal{E}_{g}|_1 = mg(70 \text{ m}), \quad P\mathcal{E}_{bc}|_1 = 0 \]  

(5)

\[ K\mathcal{E}|_2 = \frac{1}{2}mv^2, \quad P\mathcal{E}_{g}|_2 = 0, \quad P\mathcal{E}_{bc}|_2 = \frac{1}{2}k(70 \text{ m} - L)^2 \]  

(6)

Again, using (1) we get

\[ (55 \text{ kg})(9.81 \text{ m/s}^2)(70 \text{ m}) = \frac{1}{2}(55 \text{ kg})v^2 + \frac{1}{2}(0.9)(22 \text{ N/m})(70 \text{ m} - 9.67 \text{ m})^2 \]  

(7)

\[ v = 7.95 \text{ m/s} \]

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GOAL: Find new velocity when grade increases.
GIVEN: Constant power condition and grade change. Initial speed is 10 mph.

DRAW:

\[ \sum \mathbf{F} = \mathbf{F}_g - \mathbf{F}_N = \mathbf{F}_F + \mathbf{F}_T \]

\[ g \downarrow \quad mg \quad \sum \theta \]

\[ F \quad N \]

FORMULATE EQUATIONS: We’ll apply work energy:

\[ \mathcal{KE}_2 + \mathcal{PE}_2 = \mathcal{KE}_1 + \mathcal{PE}_1 + W_{nc1-2} \]  \hspace{1cm} (1)

Definition of power:

\[ P = \frac{dW}{dt} \]  \hspace{1cm} (2)

The two angles of ascent are given by

\[ \theta_1 = \arctan 0.05 = 2.86^\circ, \quad \theta_2 = \arctan 0.06 = 3.43^\circ \]

Let the speed on the initial slope be given by \( v_1 \) and on the greater slope be given by \( v_2 \).

SOLVE:

(1) \( \Rightarrow \)

\[ 0 + mgh = 0 + 0 + W_{cyclist} \]  \hspace{1cm} (3)

(2), (3) \( \Rightarrow \)

\[ P = \frac{dW_{cyclist}}{dt} = mg \frac{dh}{dt} \]  \hspace{1cm} (4)

\[ \left( \frac{dh}{dt} \right)_1 = v_1 \sin \theta_1, \quad \left( \frac{dh}{dt} \right)_2 = v_2 \sin \theta_2 \]  \hspace{1cm} (5)

(2), (4), (5) \( \Rightarrow \)

\[ P_1 = P_2 \Rightarrow v_2 = v_1 \frac{\sin \theta_1}{\sin \theta_2} \]  \hspace{1cm} (6)

\[ v_2 = 8.34 \text{ mph} \]

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4.3.21 (10 pts)

**GOAL:** Determine an electric motor's efficiency.

**GIVEN:** Mass of load, distance traveled and time needed to lift load and electrical power supplied to the motor.

**DRAW:**

![FBD Diagram](image)

**FORMULATE EQUATIONS:**
We'll determine the work done (force applied over distance), then use $P = W/t$ to find the average power and finally use $\eta = P_{out}/P_m$ to determine the efficiency.

**SOLVE:**
The work done is given by

$$W = Th = (200 \text{ kg})(9.81 \text{ m/s}^2)(4 \text{ m}) = 7848 \text{ J}$$

$$P = \frac{W}{t} = \frac{7848 \text{ J}}{3 \text{ s}} = 2616 \text{ W}$$

$$\eta = \frac{2616 \text{ W}}{3000 \text{ W}} = 0.872$$

Thus we see that $\eta = 0.872$ and we have an efficiency of 87.2 percent.


**Bonus problems**

5.1.16

**GOAL:** Determine acceleration of \( m_A \) and \( m_B \). Which one(s) accelerate is not immediately obvious.

**GIVEN:** \( T = 30 \, \text{N} \), \( m_A = 10 \, \text{kg} \), \( m_B = 1 \, \text{kg} \), \( \mu_{1a} = 0.4 \), \( \mu_{1d} = 0.35 \), \( \mu_{2a} = 0.2 \), \( \mu_{2d} = 0.15 \)

**DRAW:**

(a)

(b)

**FORMULATE EQUATIONS:**

First assume that neither block moves.

Block \( A \):

\[
0 = \left( \frac{T}{\sqrt{2}} + N_1 - m_A g \right) \tilde{J} + \left( \frac{T}{\sqrt{2}} - S_1 \right) \tilde{T}
\]

\[
\tilde{T}:
\]

\[
S_1 = \frac{T}{\sqrt{2}} = 21.2 \, \text{N}
\]

\[
\tilde{J}:
\]

\[
N_1 = -\frac{T}{\sqrt{2}} + m_A g = 76.9 \, \text{N}
\]

The maximum obtainable friction force is \( \mu_{1a} N_1 = 30.7 \, \text{N} \). All we need to keep Body \( A \) from slipping on Body \( B \) is for \( S_1 \) to be less than this and for this problem the applied force is 21.2 N. Therefore Body \( A \) doesn’t slip on Body \( B \).

Next, consider whether Bodies \( A \) and \( B \) move as a single unit. We’ll start by continuing with our assumption of no motion at all. Our new free body diagram is shown in Figure (b) and a balance of linear momentum gives us

Blocks \( A \) and \( B \):

\[
0 = \left( N_2 - (m_A + m_B) g + \frac{T}{\sqrt{2}} \right) \tilde{J} + \left( \frac{T}{\sqrt{2}} - S_2 \right) \tilde{T}
\]

\[
\tilde{T}:
\]

\[
S_2 = \frac{T}{\sqrt{2}} = 21.2 \, \text{N}
\]

\[
\tilde{J}:
\]

\[
N_2 = (m_A + m_B) g - \frac{T}{\sqrt{2}} = (11 \, \text{kg})(9.81 \, \text{m/s}^2) - \frac{30 \, \text{N}}{\sqrt{2}} = 86.7 \, \text{N}
\]

\[
S_{2_{\text{max}}} = \mu_{2a} N_2 = 0.2(86.7 \, \text{N}) = 17.3 \, \text{N}
\]

21.2 N exceeds the maximum friction force of 17.3 N and thus we will have slip between the bottom block and the floor. Thus we can now treat the two blocks as a single mass that’s slipping on the floor. Let \( x \) measure the horizontal displacement of the two blocks.

Blocks \( A \) and \( B \):

\[
(m_A + m_B) \ddot{x} \tilde{T} = \left( N_2 - (m_A + m_B) g + \frac{T}{\sqrt{2}} \right) \tilde{J} + \left( \frac{T}{\sqrt{2}} - S_2 \right) \tilde{T}
\]
\( \ddot{r}: \)

\[
\frac{(m_A + m_B)\ddot{x}}{\sqrt{2}} = \frac{T}{\sqrt{2}} - \mu_d N_2
\]

\[
(11 \text{ kg}) \ddot{x} = \frac{30 \text{ N}}{\sqrt{2}} - 0.15(86.7 \text{ N}) \Rightarrow \ddot{x} = 0.746 \text{ m/s}^2
\]

\[
\ddot{a}_A = \ddot{a}_B = 0.746 \ddot{r} \text{ m/s}^2
\]
5.1.23

**GOAL:** Explain whether the total work done by the spring on the two masses is zero or not

**GIVEN:** System configuration.

**SOLVE:** False. It is true that the forces on \( m_1 \) and \( m_2 \) are equal and opposite. These forces are acting on individual masses, the displacements and velocities of which are in general unrelated. The work done, \( \int F_1 \cdot dr_1 \) and \( \int F_2 \cdot dr_2 \) are therefore different.

**Example:** Initially the spring is extended \( L_1 - L_0 \), where \( L_0 \) is the unstretched length. If the masses are released they will move towards each other, their kinetic energies increasing in proportion to the work done by the spring.