1. \[ \vec{F} = 20 \text{N} \hat{d} - 5 \text{N} \hat{k} \]

2. \[ \vec{r}_{N_0} = 0.2 \text{m} \hat{z} + 0.3 \text{m} \hat{d} - 0.1 \text{m} \hat{k} \]

3. \[ \vec{N}_0 = \vec{r}_{N_0} \times \vec{F} \]

   \[ = (0.2 \text{m} \hat{z} + 0.3 \text{m} \hat{d} - 0.1 \text{m} \hat{k}) \times (20 \text{N} \hat{d} - 5 \text{N} \hat{k}) \]

   \[ = 4 \text{N} \text{m} \hat{k} + 1 \text{N} \text{m} \hat{d} - 1.5 \text{N} \text{m} \hat{z} + 3 \text{N} \text{m} \hat{z} \]

   \[ = 0.5 \text{N} \text{m} \hat{z} + 1 \text{N} \text{m} \hat{d} + 4 \text{N} \text{m} \hat{k} \]

(b) Moment about \( \vec{V} = 1 \hat{z} + 2 \hat{d} - 3 \hat{k} \)

\[ \vec{u}_V = \frac{\vec{V}}{||V||} = \frac{1 \hat{z} + 2 \hat{d} - 3 \hat{k}}{\sqrt{14}} \]

\[ \vec{N} = \vec{u}_V \cdot (\vec{r}_{N_0} \times \vec{F}) \]

\[ = \frac{1}{\sqrt{14}} \hat{z} + \frac{2}{\sqrt{14}} \hat{d} - \frac{3}{14} \hat{k} \cdot \left( 0.5 \text{N} \text{m} \hat{z} + 1 \text{N} \text{m} \hat{d} + 4 \text{N} \text{m} \hat{k} \right) \]

\[ = \left( \frac{0.5}{\sqrt{14}} + \frac{2}{\sqrt{14}} - \frac{12}{14} \right) \text{N} \cdot \text{m} \]

\[ = \left( \frac{9.5}{\sqrt{14}} \right) \text{N} \cdot \text{m} \]
\[ \vec{F} = 0 \]

\[ \rho (-\cos 30 \hat{x} - \sin 30 \hat{y}) - mg \hat{z} + N(\cos \theta \hat{x} + \sin \theta \hat{y}) \]

\[ \hat{x}: -\rho \cos 30 + N \cos \theta = 0 \quad \cdots \quad (1) \]

\[ \hat{y}: -\rho \sin 30 - mg + N \sin \theta = 0 \quad \cdots \quad (2) \]

From 0 to 2, solve for \( \rho \) and \( N \).
\( \hat{e}_r = \sin\theta \hat{\xi} - \cos\theta \hat{\zeta} \)
\( \hat{e}_\theta = \cos\theta \hat{\xi} + \sin\theta \hat{\zeta} \)

\[\Sigma F_y = 0\]
\[A_y = 50N\]
\[\Sigma F_x = 0\]
\[A_x = 0\]
\[\Sigma \dot{F}_N = \ddot{\omega} + \dot{R} = 0\]
\[\Sigma \dot{M}_N = -\left( \frac{2l}{3} - \frac{l}{2} \right) \hat{e}_r \times \omega \hat{\zeta} - \frac{2l}{3} \hat{e}_r \times \dot{R}\]

\( x_0 = 30 \text{ cm} \quad k = 1 \text{ N/mm} \)
\[\Sigma F = 0\]
\[A_x \hat{\xi} + A_y \hat{\zeta} - \omega \hat{\zeta} + F_s \hat{\zeta} = 0\]
\[\Sigma \dot{M}_A = 0\]
\[\frac{l}{2} \hat{e}_r \times \omega \hat{\zeta} + l \hat{e}_r \times F_s \hat{\zeta} = 0\]

\[F_s = \frac{kx}{s} \quad x = \frac{F_s}{k}\]

Compressed length: \( x_0 - x = \)
Distance of pt S = Compressed length of spring + 1000

Eq. for a couple system @ $\frac{4l}{5}$

\[ 2F = 0 \]

\[ A_y - \omega + F_5 = 0 \]

\[ F_{net} = 0 \]

\[ \sum_{i=2}^{\infty} \hat{e}_r \times A_y \hat{j} = \omega \]

\[ + \left( \frac{4l}{5} - \frac{L}{2} \right) \hat{e}_r \times \omega \hat{j} \]

\[ + \frac{1}{5} \hat{e}_r \times F_5 \hat{j} \]
\[ F_{AB} \]

\[ F_x = m \ddot{x} \]

\[ 5T = m \ddot{x}_A \quad \text{(1)} \]

\[ \ell_{bat} = -5\ddot{x}_A + 4\ddot{x}_B \]

\[ \dot{\ell}_{bat} = -5\ddot{x}_A + 4\ddot{x}_B = 0 \quad \text{(3)} \]

\[ \dot{\ell}_{bot} = -5\ddot{x}_A + 4\ddot{x}_B = 0 \quad \text{(3)} \]

\[ 3 \text{ Eqns in } 3 \text{ unknowns} \]
**Solve** We know from our analysis of Figure 2.70 that if the free end of the rope is pulled at a speed \(v\), then the left pulley (and consequently the movable platform) will rise at a speed \(\frac{1}{3}\). Thus, because the rope is being pulled at 0.90 m/s, the left pulley and platform will rise at 0.45 m/s, and so the time needed to reach the treehouse is

\[
t = \frac{6 \text{ m}}{0.45 \text{ m/s}} = 13 \frac{1}{3} \text{ s}
\]

**Check** This pulley is so simple that there's no problem checking consistency—when the right end is pulled down the left will go up.

---

**Example 2.22 Double Pulley**

Repeat Example 2.21 for the pulley system shown in Figure 2.73.

- **Goal** Find the time needed to reach the treehouse.
- **Given** Pulley arrangement and speed of pulley rope.
- **Draw** Figure 2.74 shows our system.

**Assume** No additional assumptions are needed.

**Formulate Equations** We will be using (2.76) a couple of times.

**Solve** A schematic of this more complicated pulley system is shown in Figure 2.74a. As shown in Figure 2.74b, the system can be regarded as being made up of two of the simpler pulley systems introduced in Figure 2.69. Although the two-pulley system on the right in Figure 2.74b...
seems more involved than the one-pulley system on the left, the two systems are actually equivalent. Lifting end $A$ an amount $L$ will raise pulley $3$ an amount $\frac{L}{2}$, and pulling down an amount $L$ on end $B$ will raise pulley $2$ by $\frac{L}{2}$.

So how do we approach this? The same way we did previously: by “conservation of rope.” I’ve drawn in coordinates $x_1, x_2, x_3$ in Figure 2.74a to quantify everything that’s moving in the system. Invoking the conservation of rope principle, we obtain two relationships:

\[
\Delta x_1 + 2\Delta x_2 = 0 \\
-\Delta x_2 + 2\Delta x_3 = 0
\]

and thus can obtain

\[
\dot{x}_1 + 2\dot{x}_2 = 0 \\
-\dot{x}_2 + 2\dot{x}_3 = 0
\]

Solving for $\dot{x}_1$ and $\dot{x}_2$ yields

\[
\dot{x}_2 = -\frac{1}{2}\dot{x}_1 \\
\dot{x}_3 = \frac{1}{2}\dot{x}_2 = -\frac{1}{4}\dot{x}_1
\]

(2.78)  
(2.79)

The platform rises up at only one-fourth the speed at which the rope’s being pulled. By pulling down at $0.90$ m/s, your assistant caused the platform to move up at $0.225$ m/s. The time it takes now to reach the treehouse is

\[
\frac{6.0 \text{ m}}{0.225 \text{ m/s}} = 26.7 \text{ s}
\]

**Check** The result makes physical sense. If you pull down the right rope with speed $v$, you can see that the middle pulley will rise at half that rate: $\frac{v}{2}$. Then, as the middle pulley must rise at half the rate that its right rope is moving, it naturally moves up at one-quarter the speed of your original pull rate: $\frac{v}{4}$.

When we get into forces, we will revisit these pulley problems and see whether the forces change as well. After all, if it’s taking twice as long to reach the treehouse, there must be some payoff somewhere; otherwise why would people have invented all those complicated pulleys that can be seen around the world? The answer, not surprisingly, is that, although it takes longer to move the load, you don’t have to exert as much force.

---

**EXERCISES 2.5**

2.5.1. A secret agent is running toward the back of a moving bus, his intent being to jump off the back before the explosive he planted at the front goes off. At the illustrated instant, the bus has a constant velocity of $v_{bus} = 10t$ mph. Currently, our agent is 30 feet from the rear of the bus and moving at zero mph with respect to the bus.

- **a.** What constant acceleration will he need with respect to the bus so that he will leave the rear of the bus with zero velocity relative to the ground?
- **b.** Where will he land with respect to point $A$?
7.3.23  
**GOAL:** Find the support loads at $B$.  
**GIVEN:** We are given the geometry of the system, the applied load at the end of the pole, that the weight of the pole is negligible, and that the system is in equilibrium.  
**ASSUME:** We assume the system is nonplanar.  
**DRAW:** We draw an FBD of the pole.

![FBD Diagram](image)

**FORMULATE EQUATIONS AND SOLVE:** First, we construct the force vector acting on the pole at $A$. 

$$ T = T \frac{\mathbf{r}_{AC}}{||\mathbf{r}_{AC}||} = \frac{100}{7} (-2i - 6j + 3k) \text{ N} = (-28.6i - 85.7j + 42.9k) \text{ N} $$

We now apply the vector force equilibrium condition.

$$ \sum \mathbf{F} = 0 $$

$$ F_B + T = 0 $$

$$ F_B + \frac{100}{7} (-2i - 6j + 3k) \text{ N} = 0 $$

$$ F_B = \frac{100}{7} (2i + 6j - 3k) \text{ N} = (28.6i + 85.7j - 42.9k) \text{ N} $$

$$ [F_B = (28.6i + 85.7j - 42.9k) \text{ N}, \ ||F_B|| = 100 \text{ N}] \quad \text{support force at B} $$

Note that the magnitude of the support force at $B$ is exactly equal to the applied load at $A$, as expected.

We next apply the vector moment equilibrium condition about $B$. With the position vector

$$ \mathbf{r}_{BA} = (5i + 6j + k) \text{ m} $$

the vector moment equilibrium condition about $B$ is

$$ \sum \mathbf{M}_B = 0 $$
\[ M_B + r_{BA} \times T = 0 \]  

We now need to calculate the cross product:

\[
r_{BA} \times T = \frac{100}{7} \begin{vmatrix} i & j & k \\ 5 & 6 & 1 \\ -2 & -6 & 3 \end{vmatrix} N \cdot m = (343i - 243j - 257k) N \cdot m
\]

Substituting this result into (1), we have

\[
M_B + (343i - 243j - 257k) N \cdot m = 0
\]

\[
M_B = (-343i + 243j + 257k) N \cdot m
\]

\[
\boxed{M_B = (-343i + 243j + 257k) N \cdot m, \quad \|M_B\| = 493 N \cdot m} \quad \text{support moment at B}
\]

**CHECK:** We can check our answer by verifying that the net moment about any other point is zero. In particular, we will verify that the moment about \( A \) is zero. Further, since we already know the components of the forces, we can easily apply the scalar momentum equilibrium conditions. Applying the moment equilibrium condition about the \( x \)-axis at \( A \), we have

\[
\sum M_{x \theta A} = 0
\]

\[
M_{Bx} + (1 \text{ m})B_y + (6 \text{ m})B_z = 0
\]

\[-343 N \cdot m + (1 \text{ m})(85.7 N) - (6 \text{ m})(-42.9 N) = 0
\]

\[
0 = 0
\]

About the \( y \)-axis at \( B \), we have

\[
\sum M_{y \theta B} = 0
\]

\[
M_{By} + (5 \text{ m})B_z - (1 \text{ m})B_x = 0
\]

\[243 N \cdot m + (5 \text{ m})(-42.9 N) - (1 \text{ m})(28.6 N) = 0
\]

\[
0 = 0
\]

Finally, about the \( z \)-axis at \( B \), we have

\[
\sum M_{z \theta B} = 0
\]

\[
M_{Bz} - (6 \text{ m})B_x - (5 \text{ m})B_y = 0
\]

\[257 N \cdot m + (6 \text{ m})(28.6 N) - (5 \text{ m})(85.7 N) = 0
\]

\[
0 = 0
\]

Since the net moment about \( B \) is zero in all three coordinate directions, our answer must be correct.