SAMPLE 3.1 A mass and a pulley. A block of mass $m$ is held up by applying a force $F$ through a massless pulley as shown in the figure. Assume the string to be massless. Draw free-body diagrams of the mass and the pulley separately and as one system.

Solution The free-body diagrams of the block and the pulley are shown in Fig. 3.11. Since the string is massless and we assume an ideal massless pulley, the tension in the string is the same on both sides of the pulley. Therefore, the force applied by the string on the block is simply $F$. When the mass and the pulley are considered as one system, the force in the string on the left side of the pulley doesn’t show because it is internal to the system.

Figure 3.11: The free-body diagrams of the mass, the pulley, and the mass-pulley system. Note that for the purpose of drawing the free-body diagram we need not show that we know that $R = 2F$. Similarly, we could have chosen to show two different rope tensions on the sides of the pulley and reasoned that they are equal as is done in the text.
SAMPLE 3.2 Forces in strings. A block of mass $m$ is held in position by strings $AB$ and $AC$ as shown in Fig. 3.13. Draw a free-body diagram of the block and write the vector sum of all the forces shown on the diagram. Use a suitable coordinate system.

Solution To draw a free body diagram of the block, we first free the block. We cut strings $AB$ and $AC$ very close to point $A$ and show the forces applied by the cut strings on the block. We also isolate the block from the earth and show the force due to gravity. (See Fig. 3.14.)

To write the vector sum of all the forces, we need to write the forces as vectors. To write these vectors, we first choose an $xy$ coordinate system with basis vectors $\hat{i}$ and $\hat{j}$ as shown in Fig. 3.14. Then, we express each force as a product of its magnitude and a unit vector in the direction of the force. So,

$$\vec{T}_1 = T_1 \frac{\vec{r}_{AB}}{\| \vec{r}_{AB} \|},$$

where $\vec{r}_{AB}$ is a vector from $A$ to $B$ and $\| \vec{r}_{AB} \|$ is its magnitude. From the given geometry,

$$\vec{r}_{AB} = -2m\hat{i} + 2m\hat{j}\quad\Rightarrow\quad \hat{r}_{AB} = \frac{2m\hat{i} - \hat{j}}{\sqrt{4 + 1}} = \frac{1}{\sqrt{5}}(-\hat{i} + \hat{j}).$$

Thus,

$$\vec{T}_1 = T_1 \frac{1}{\sqrt{5}}(-\hat{i} + \hat{j}).$$

Similarly,

$$\vec{T}_2 = T_2 \frac{1}{\sqrt{5}}(\hat{i} + 2\hat{j})$$

$$mg\vec{g} = -mg\hat{j}.$$ 

Now, we write the sum of all the forces:

$$\sum \vec{F} = \vec{T}_1 + \vec{T}_2 + mg\vec{g}$$

$$= \left( \frac{T_1}{\sqrt{5}} + \frac{T_2}{\sqrt{5}} \right)\hat{i} + \left( \frac{T_1}{\sqrt{5}} + \frac{2T_2}{\sqrt{5}} - mg \right)\hat{j}.$$ 

The $\hat{i}$ and $\hat{j}$ components of the net force depend on the values of the scalars (magnitudes) $T_1$, $T_2$ and $mg$. 

$$\sum \vec{F} = \left( \frac{T_1}{\sqrt{5}} + \frac{T_2}{\sqrt{5}} \right)\hat{i} + \left( \frac{T_1}{\sqrt{5}} + \frac{2T_2}{\sqrt{5}} - mg \right)\hat{j}$$
SAMPLE 3.3 Two bodies connected by a massless spring. Two carts \( A \) and \( B \) are connected by a massless spring. The carts are pulled to the left with a force \( F \) and to the right with a force \( T \) as shown in Fig. 3.16. Assume the wheels of the carts to be massless and frictionless. Draw free body diagrams of

- cart \( A \),
- cart \( B \), and
- carts \( A \) and \( B \) together.

Solution The three free body diagrams are shown in Fig. 3.15 (a) and (b). In Fig. 3.15 (a) the force \( F_g \) is applied by the spring on the two carts. Why is this force the same on both carts? In Fig. 3.15(b) the spring is a part of the system. Therefore, the forces applied by the spring on the carts and the forces applied by the carts on the spring are internal to the system. Therefore these forces do not show on the free body diagram.

Note that the normal reaction of the ground can be shown either as separate forces on the two wheels of each cart or as a resultant reaction.
SAMPLE 3.4 Two carts connected by pulleys. The two masses shown in Fig. 3.17 have frictionless bases and round frictionless pulleys. The inextensible massless cord connecting them is always taut. Mass A is pulled to the left by force $F$ and mass B is pulled to the right by force $P$ as shown in the figure. Draw free body diagrams of each mass.

Solution Let the tension in the cord be $T$. Since the pulleys and the cord are massless, the tension is the same in each section of the cord. This equality is clearly shown in the Free body diagrams of the two masses below.

![Free body diagrams of the two masses](image)

Figure 3.19: Free body diagrams of the two masses.

Comments: We have shown unequal normal reactions on the wheels of mass B. In fact, the two reactions would be equal only if the forces applied by the cord on mass B satisfy a particular condition. Can you see what condition must be satisfy for, say, $N_{A_1} = N_{A_2}$?

[Hint: think about the moment balance about the center-of-mass A.]

SAMPLE 3.5 Structures with pin connections. A horizontal force $T$ is applied on the structure shown in the figure. The structure has pin connections at A and B and a roller support at C. Bars AB and BC are rigid. Draw free body diagrams of each bar and the structure including the spring.

Solution The free body diagrams are shown in figure 3.20. Note that there are both vertical and horizontal forces at the pin connections because pins restrict translation in any direction. At the roller support at point C there is only vertical force from the support ($T$ is an externally applied force).

![Free body diagrams of (a) the individual bars and (b) the structure as a whole](image)

Figure 3.20: Free body diagrams of (a) the individual bars and (b) the structure as a whole.
SAMPLE 3.6 The four bar linkage shown in the figure is pushed to the right with a force $F$ as shown in the figure. Pins A, C & D are frictionless but joint B is rusty and has friction. Neglect gravity; and assume that bar AB is massless. Draw free body diagrams of each of the bars separately and of the whole structure. Use consistent notation for the interaction forces and moments. Clearly mark the action-reaction pairs.

Solution A ‘good’ pin resists any translation of the pinned body, but allows free rotation of the body about an axis through the pin. The body reacts with an equal and opposite force on the pin. When two bodies are connected by a pin, the pin exerts separate forces on the two bodies. Ideally, in the free-body diagram, we should show the pin, the first body, and the second body separately and draw the interaction forces. This process, however, results in too many free body diagrams. Therefore, usually, we let the pin be a part of one of the objects and draw the free body diagrams of the two objects.

Note that the pin at joint B is rusty, which means, it will resist a relative rotation of the two bars. Therefore, we show a moment, in addition to a force, at point B of each of the two rods AB and BC.

Figure 3.21: Style 1: Free body diagrams of the structure and the individual bars. The forces shown in (a) and (b) are the same.

Figure 3.21 shows the free body diagrams of the structure and the individual rods. In this figure, we show the forces in terms of their x- and y-components. The directions of the forces are shown by the arrows and the magnitude is labeled as $A_x$, $A_y$, etc. Therefore, a force, shown as an arrow in the positive x-direction with ‘magnitude’ $A_x$, is the same as that shown as an arrow in the negative x-direction with magnitude $-A_x$. Thus, the free body diagrams in Fig. 3.21(a) show exactly the same forces as in Fig. 3.21(b).

In Fig. 3.22, we show the forces by an arrow in an arbitrary direction. The corresponding labels represent their magnitudes. The angles represent the unknown directions of the forces.
Figure 3.22: Style 2: Free body diagrams of the structure and the individual bars. The forces shown in (a) and (b) are the same.

In Fig. 3.24, we show yet another way of drawing and labeling the free body diagrams, where the forces are labeled as vectors.

Figure 3.24: Style 3: Free body diagrams of the structure and the individual bars. The label of a force indicates both its magnitude and direction. The arrows are arbitrary and merely indicate that a force or a moment acts on those locations.

Note: There are no two-force bodies in this problem. Bar AB is massless but is not a two-force member because it has a couple at its end.
Most often, we are interested in cases where the contacting bodies have some non-zero relative angular velocity — a ball sitting still on level ground may be technically in rolling contact, but not interestingly so.

The simplest common example is the rolling of a round wheel on a flat surface in two dimensions. See figure 3.30.

![FBD of Wheel](image)

Figure 3.30: Pure rolling of a round wheel on a flat slope in two dimensions.

In practice, there is often confusion about the direction and magnitude of the force \( F \) shown in the free body diagram in figure 3.30. Here is a recipe:

1.) Draw \( F \) as shown in any direction which is tangent to the surface.

2.) Solve the statics or dynamics problem and find the scalar \( F \). (It may turn out to be a negative, which is fine.)

3.) Check that rolling is really possible; that is, that slip would not occur. If the force is greater than the frictional strength, \( |F| > \mu N \), the assumption of rolling contact is not appropriate. In this case, you must assume that \( F = \mu N \) or \( F = -\mu N \) and that slip occurs; then, re-solve the problem.

![FBD of Ball](image)

Figure 3.31: Rolling ball in 3-D. The force \( \vec{F} \) and moment \( \vec{M} \) are applied loads from, say, wind, gravity, and any attachments. \( N \) is the normal reaction and \( F_1 \) and \( F_2 \) are the in plane components of the frictional reaction. One must check the no-slip condition, \( \mu N^2 \geq F_1^2 + F_2^2 \).
SAMPLE 3.7 Stacked blocks at rest on an inclined plane. Blocks $A$ and $B$ with masses $m$ and $M$, respectively, rest on a frictionless inclined surface with the help of force $T$ as shown in Fig. 3.35. There is friction between the two blocks. Draw free body diagrams of each of the two blocks separately and a free body diagram of the two blocks as one system.

Solution The three free body diagrams are shown in Fig. 3.37 (a) and (b). Note the action and reaction pairs between the two blocks; the normal force $N_A$ and the friction force $F_f$ between the two bodies $A$ and $B$. If we consider the two blocks together as a system, then the forces $N_A$ and $F_f$ do not show on the free body diagram of the system (see Fig. 3.37(b)), because now they are internal to the system.

Figure 3.36: Two blocks slide down a frictional inclined plane. The blocks are connected by a light rigid rod.

Figure 3.37: Free body diagrams of (a) block $A$ and block $B$ separately and (b) blocks $A$ and $B$ together.

SAMPLE 3.8 Two blocks slide down a frictional inclined plane. Two blocks of identical mass but different material properties are connected by a massless rigid rod. The system slides down an inclined plane which provides different friction to the two blocks. Draw free body diagrams of the two blocks separately and of the system (two blocks with the rod).

Solution The free body diagrams are shown in Fig. 3.38. Note that the friction forces on the two blocks are different because the coefficients of friction are different for the two blocks. The normal reaction of the plane, however, is the same for each block (why?).

Figure 3.38: Free body diagrams of (a) the two blocks and the rod as a system and (b) the two blocks separately.
SAMPLE 3.9  Massless pulleys. A force $F$ is applied to the pulley arrangement connected to the cart of mass $m$ shown in Fig. 3.41. All the pulleys are massless and frictionless. The wheels of the cart are also massless but there is friction between the wheels and the horizontal surface. Draw a free body diagram of the cart, its wheels, and the two pulleys attached to the cart, all as one system.

Solution  The free body diagram of the cart system is shown in Fig. 3.39. The force in each part of the string is the same because it is the same string that passes over all the pulleys.

![Figure 3.39: Free-body diagram of the cart.](image)

SAMPLE 3.10  A unicyclist in action. A unicyclist weighing 160 lbs exerts a force on the front pedal with a vertical component of 30 lbf at the instant shown in figure 3.42. The rear pedal barely touches the other foot. Assume the wheel and the frame are massless. Draw free body diagrams of the cyclist and the cycle. Make other reasonable assumptions if required.

Solution  Let us assume, there is friction between the seat and the cyclist and between the pedal and the cyclist’s foot. Let’s also assume a 2-D analysis. The free body diagrams of the cyclist and the cycle are shown in Fig. 3.40. We assume no couple interaction at the seat.

![Figure 3.42: The unicyclist](image)

Figure 3.40: Free-body diagram of the cyclist and the cycle.