A method called the eigensystem realization algorithm is developed for modal parameter identification and model reduction of dynamic systems from test data. A new approach is introduced in conjunction with the singular-value decomposition technique to derive the basic formulation of minimum order realization which is an extended version of the Ho-Kalman algorithm. The basic formulation is then transformed into modal space for modal parameter identification. Two accuracy indicators are developed to quantitatively identify the system and noise modes. For illustration of the algorithm, an example is shown using experimental data from the Galileo spacecraft.

**Introduction**

The state space model has received considerable attention for system analyses and design in recent control and systems research programs. One of these areas, in particular, is control of large space structures. In order to design controls for a dynamic system it is necessary to have a mathematical model that will adequately describe the system's motion. The process of constructing a state space representation from experimental data is called system realization.

During the past two decades, numerous algorithms for the construction of state space representations of linear systems have appeared in the controls literature. Among the first were the works of Gilbert and Kalman, introducing the important principles of realization theory in terms of the concepts of controllability and observability. Both techniques use the transfer function matrix to solve the realization problem. Ho and Kalman approached this problem from a new viewpoint. They showed that the minimum realization problem is equivalent to a representation problem involving a sequence of real matrices known as Markov parameters (pulse response functions). By minimum realization is meant a model with the smallest state space dimension among systems realized that has the same input-output relations within a specified degree of accuracy. Questions regarding the minimum realization from various types of input-output data and the generation of a minimum partial realization are studied by Tether, Silverman, and Rossen and Lapidus using Markov parameters. Rossen and Lapidus successfully applied Ho-Kalman and Tether methods to chemical engineering systems. A common weakness of the preceding schemes is that effects of noise on the data analysis were not evaluated. Zeiger and McEwen proposed a combination of the Ho-Kalman algorithm with the singular-value decomposition technique for the treatment of noisy data. However, no theoretical or numerical studies were reported by Zeiger and McEwen. Among follow-up developments along similar lines, Kung presented another algorithm in conjunction with the singular-value decomposition technique to incorporate the presence of the noise. Note that the singular-value decomposition technique has been widely recognized as being very effective and numerically stable. Although several techniques of minimum realization are available in the literature, formal direct application to modal parameter identification for flexible structures has not been addressed.

In the structures field, the finite element technique is used almost exclusively for constructing analytical models. This approach is well established and normally provides a model accurate enough for structural design purposes. Once the structure is built, static and dynamic tests are performed. These test results are used to refine the finite element model, which is then used for final analysis. The traditional approach to analytical model development may not be accurate enough for use in designing a vibration control system for flexible structures. Another approach is to realize a model directly from the experimental results. This requires the construction of a minimum-order model from the test data that characterizes the dynamics of the system at the selected control and measurement positions. The present state-of-the-art in structural modal testing and data analysis is one of controversy about the best technique to use. Classical test techniques, which may provide only good frequency and moderate mode shape accuracy, are often considered adequate for finite element model verification purposes. On the other hand, advanced data analysis techniques that offer significant reductions in test time and improved accuracy have been available. However, they are not yet fully accepted. For example, Vold and Russell presented a method using frequency-response functions and time-domain analysis for direct identification of modal parameters including repeated eigenvalues. A comparison of contemporary methods using data from the Galileo spacecraft test is provided by Chen.

Although structural dynamics techniques are generally successful for ground data, further incorporation with work from the controls discipline is needed to solve modal parameter identification/control problems. For example, it is known from control theory that a system with repeated eigenvalues and independent mode shapes is not identifiable by single input and single output. Methods which allow only one initial condition (input) at a time will miss repeated eigenvalues. Also, if the realized system is not of minimum order and matrix inversion is used for constructing an oversized state matrix, numerical errors may become dominant.

Under the interaction of structure and control disciplines, the objective of this paper is to introduce an eigensystem realization algorithm (ERA) for modal parameter identification and model reduction for dynamical systems from test data. The algorithm consists of two major parts, namely, basic formulation of the minimum-order realization and modal parameter identification. In the section of the basic formulation, the Hankel matrix, which represents the data structure for the Ho-Kalman algorithm, is generalized to allow ran-
dom distribution of Markov parameters generated by free decay responses. A unique approach based on this generalized Hankel matrix is developed to extend the Ho-Kalman algorithm in combination with the singular-value decomposition technique. Through the use of the generalized Hankel matrix, a linear model is realized for a dynamical system model phase collinearity, are developed to quantify the system damping properties. The problem of system realization is as follows: Given the measurement functions \( y(k) \), construct the integers. For the system with initial state response measurements, simply replace \( H_n(k-1) \) by \( H_n(k) \). Now observe that

\[
H_n(k) = V_r A^k W_s; \quad V_r = \begin{bmatrix} C \\ CA^l \\ \vdots \end{bmatrix}
\]

and

\[
W_s = [B, A^l B, \ldots, A^{l-1} B]
\]

where \( V_r \) and \( W_s \) are the observability and controllability matrices, respectively. Note that \( V_r \) and \( W_s \) are rectangular matrices with dimensions \( rp \times n \) and \( n \times ms \), respectively. Assume that there exists a matrix \( H^* \) satisfying the relation

\[
W_s H^* V_r = I_n
\]

where \( I_n \) is an identity matrix of order \( n \). It will be shown that the matrix \( H^* \) plays a major role in deriving the ERA. What is \( H^* \)? Observe that, from Eqs. (5) and (6),

\[
H_n(0) H^* H_n(0) = V_r W_s H^* V_r = V_r W_s = H_n(0)
\]

The matrix \( H^* \) is thus the pseudoinverse of the matrix \( H_n(0) \) in a general sense. For a single input, there exists a case [see Eq. (5)] where the rank of \( H_n(0) \) equals the column number of \( H_n(0) \), then

\[
H^* = [H_n(0)]^T H_n(0) - \frac{1}{[H_n(0)]^T}
\]

On the other hand, there exists a case for a single output [see Eq. (5)] where the rank equals the row number, then

\[
H^* = [H_n(0)]^T [H_n(0) [H_n(0)]^T] - \frac{1}{i}
\]

The matrix \( H_n(1) H^* \) has been used in the structural dynamics field to identify system modes and frequencies. This is a special case representing a single input which cannot realize a system that has repeated eigenvalues, or a noise-free system unless the system order is known a priori. More discussion can be found in the Appendix.

A general solution for \( H^* \) is given below. For an \( n \)-th-order system, find the nonsingular matrices \( P \) and \( Q \) such that

\[
H_n(0) = PDQ^T
\]

where the \( rp \times n \) matrix \( P \) and the \( ms \times n \) matrix \( Q \) are isometric matrices (all of the columns are orthonormal, leaving the singular values of \( H_n(0) \) in the diagonal matrix \( D \) with positive elements \( |d_1, d_2, \ldots, d_m| \). The rank of \( H_n(0) \) is determined by testing the singular values for zero (relative to desired accuracy) which will be described in the next section. Define

\[
H_n(0) = PDQ^T = [PD] [Q^T] = P_d Q^T
\]

Each of the four matrices \([P_d^T, Q^T, W_s, V_r][\) has rank and row number \( n \). By Eq. (5) with \( k = 0 \),

\[
V_r W_s = H_n(0) = P_d Q^T
\]

Multiplying on the left by \( P_d^T \) and solving for \( Q^T \) yields

\[
TW_s = (P_d^T P_d)^{-1} P_d^T V_r W_s = Q^T
\]
The matrix $T$ is nonsingular because if
$$U = W_i Q (Q^T Q)^{-1} W_i Q$$
then $TU = I$ by Eq. (13). Since $TU = I = UT$ for nonsingular $T$ and $U$, then
$$W_i [Q P_{ij}^{-1} P_{ij}^T] V_r = I_n$$
(14)
Hence, by Eq. (14),
$$H' = [Q] [P_{i}^{T} P_{i}^{-1} P_{i}^T] = [Q] [D^{-1} P^{-1}] = QP_{i}^T$$
(15)
The dimension of matrices $Q$ and $P_{i}$ with rank $n$ are $m \times n$ and $n \times r$, respectively. Define $0_{n}$ as a null matrix of order $r$, $I_{m}$ an identity matrix, $E_{m}^r = [I_{m}, 0_{m}, \ldots, 0_{m}]$, and $E_{m}^p = [I_{m}, 0_{m}, 1]$. With the aid of Eqs. (5), (6), and (15), a minimum order-realization can be obtained from
$$Y(k+1) = E_{p}^{T} H_{n} (k) E_{m} = E_{p}^{T} V_{r} H_{p} A_{k} W_{e} E_{m}$$
$$= E_{p}^{T} V_{r} H_{p} (0) QP_{d}^{T} V_{r} A^{k} W_{e} E_{m}$$
$$= E_{p}^{T} H_{n} (0) QP_{d}^{T} V_{r} A^{k} W_{e} P_{p} H_{n} (0) E_{m}$$
$$= E_{p}^{T} P_{d} [P_{p} H_{n} (0) Q]^{1/2} P^{-1} T E_{m}$$
$$= E_{p}^{T} P_{d} [D^{-1/2} P^{T} H_{n} (1) QD^{-1/2}]^{1/2} [D^{-1/2} P^{T} E_{m}$$
(16)
This is the basic formulation of realization for the ERA. The triple $[D^{-1/2} P^{T} H_{n} (1) QD^{-1/2}]^{1/2}$ is a minimum realization since the order $n$ of $P_{p} H_{n} (1) Q$ equals the dimension of the state vector $x$. The same solution, in a different form, for the case where $j_i = t_i = 1$ can be obtained by a completely different approach as shown in Refs. 3 and 19. The system [Eqs. (1) and (2)] with this realization is written as
$$x(k+1) = D^{-1/2} P^{T} H_{n} (1) QD^{-1/2} x(k) + D^{1/2} Q^{T} E_{m}$$
(17)
$$y(k) = E^{T} P D^{1/2} Q^{T} x(k)$$
(18)
where
$$x(k) = W_{i} QD^{-1/2} x(k)$$
(19)
Now, the case can be summarized as follows.
A finite dimensional, discrete-time, linear time-invariant dynamical system with multi-input and multi-output is realizable in terms of the measurement functions if the system is controllable and observable (the ranks of matrices $V$, and $W$, are $n$). A simple exercise, such as replacing $Y(k+1)$ by $Y(k)$ in Eq. (16), shows that the algorithm developed above is also true for the realization of a system with initial state response.
Note that no restrictions on system eigenvalues are given for this case. In other words, this technique can realize a system with repeated eigenvalues. As byproducts of this approach, two alternative algorithms identified as (A1) and (A2) are derived in the Appendix.

Modal Parameter Identification and Model Reduction

The presence of almost unavoidable noise and structural nonlinearity introduces uncertainty about the rank of the generalized Hankel matrix and, hence, about the dimension of the resulting realization. By employing the singular-value decomposition (SVD) technique, the rank structure of the Hankel matrix can be displayed quantitatively. The set of singular values can be used to judge the distance of the matrix with determined order to a lower-order one. Therefore, the structure of the generalized Hankel matrix can be properly explored to solve the realization problem efficiently. These include an excellent numerical performance, stability of the realization, and flexibility in determining order-error tradeoff.

Assume that, by Eq. (10),
$$D = \text{diag} \{d_1, d_2, \ldots, d_n, d_{n+1}, \ldots, d_N \}$$
(20)
with
$$d_1 \geq d_2 \geq \ldots \geq d_n \geq d_{n+1} \geq \ldots \geq d_N$$
(21)
If the matrix $H_{n}(0)$ has rank $n$ then all of the singular values $d_i(i = n+1, \ldots, N)$ should be zero. When singular values $d_i(i = n+1, \ldots, N)$ are not exactly zero but very small, then one can easily recognize that the matrix $H_{n}(0)$ is not far away from an $n$-rank matrix. However, there can be real difficulties in determining a gap between the computed last nonzero singular value and what effectively should be considered zero, when measurement noise is present. Possible sources of the noise can be attributed to the measurement signal, computer roundoff, and instrument imperfections.

Look at the singular value $d_n$ of the matrix $H_{n}(0)$. Choose a number $\delta$ based on measurement errors incurred in estimating the elements of $H_{n}(0)$ and/or roundoff errors incurred in a previous computation to obtain them. If $\delta$ is chosen as a "zero threshold" such that $\delta \leq d_n$, then the matrix $H_{n}(0)$ is considered to have rank $n$. Unless information about the certainty of the measurement data is given, the number $\delta$ is defined as a function of the precision limit in the computer. For example, $\delta = d_n / d_1$ cannot exceed the precision limit; further details are found in Ref. 11.

After the test of singular values, assume that the matrix $[D^{-1/2} P^{T} H_{n} (k) Q D^{-1/2}]$ has rank $n$. Find the eigenvalues $\lambda$ and eigenvectors $\psi$ such that
$$\psi^{-1} [D^{-1/2} P^{T} H_{n} (k) Q D^{-1/2}] \psi = \lambda$$
(22)
The modal damping rates and damped natural frequencies are simply the real and imaginary parts of the eigenvalues, after transformation from the $z$ to the $s$ plane using the relationship
$$s = [(o \pm j \omega) / (\Delta \Omega)] \quad i = \sqrt{-1}$$
(23)
where $\Delta \Omega$ is the data sampling interval and $j$ is an integer. The integer $k$ is generally chosen as 1 for simplicity. Although $\lambda$ and $\psi$ are complex numbers, computations of Eq. (22) can be performed using a real algorithm since the state matrix realized for flexible structures has independent eigenvectors.

The triple $\{ \psi^{-1} D^{1/2} Q^{T} E_{m}, E_{m}^{T} P D^{1/2} \}$ is obviously a minimum order realization simply by observing Eq. (16). The matrix $E_{m}^{T} P D^{1/2} \psi$ is called mode shapes and the matrix $\psi^{-1} D^{1/2} Q^{T} E_{m}$ initial modal amplitudes. To quantify the system and noise modes, two indicators are developed as follows.

Modal Amplitude Coherence $\gamma$

If the information about the uncertainties of the measurement is minimum, the rank thus determined by the SVD becomes larger than the number of excited and observed system modes to represent the presence of noises in modal space. In modal parameter identification, the indicator referred to as modal amplitude coherence is developed to quantitatively distinguish the system and noise modes. Based on the accuracy parameter, the degree of modal excitation (controllability) is estimated.

The modal amplitude coherence is defined as the coherence between each modal amplitude history and an ideal one
formed by extrapolating the initial value of the history to latter points using the identified eigenvalue. Let the control input matrix (initial condition) be expressed as

$$\psi^{-1}D^{\theta}Q^TE_m = [b_1, b_2, ..., b_n]^*$$ (24)

where the asterisk means transpose complex conjugate, and the $1 \times m$ column vector $b_j$ corresponds to the system eigenvalue $s_j (j = 1, ..., n)$. Consider the sequence

$$q_j = [b_j, \exp(t_1 \Delta s_j), b_j, ..., \exp(t_{m-1} \Delta s_j), b_j]$$ (25)

which represents the ideal modal amplitude in the complex domain containing information of the magnitude and phase angle with time step $\Delta t$. Now, define vector $q_j$ such that

$$\psi^{-1}D^{\theta}Q^T = [q_1, q_2, ..., q_n]^*$$ (26)

The complex vector $q_j$ represents the modal amplitude time history from the real measurement data obtained by the decomposition of the Hankel matrix. Let $\gamma_j$ be defined as the coherence parameter for the $j$th mode, satisfying the relation

$$\gamma_j = \left| q_jq_j^T \right| / \left( q_jq_j^T + q_jq_j^T \right)$$ (27)

where $| |$ represent the absolute value. The parameter $\gamma_j$ can have only the values between 0 and 1. $\gamma_j = 1$ indicates that the realized system eigenvalue $s_j$ and the initial modal amplitude $b_j$ are very close to the true values for the $j$th mode of the system. On the other hand, if $\gamma_j$ is far away from the value 1, the $j$th mode is a noise mode. However, to make a clear distinction between the system and noise modes requires further study. Obviously, the parameter $\gamma_j$ quantifies the degree to which the modes were excited by a specific input, i.e., the degree of controllability.

**Modal Phase Collinearity $\mu_j$**

For lightly damped structures, normal mode behavior should be observed. An indicator referred to as the modal phase collinearity is developed to measure the strength of the linear functional relationship between the real and imaginary parts of the mode shape for each mode. Based on the accuracy indicators, the degree of modal observation is estimated. Define

$$E_p^T P D^\theta \psi = [c_1, c_2, ..., c_n]$$ (28)

where $c_j (j = 1, 2, ..., n)$ is the mode shape corresponding to the $j$th realized mode. Let the column vector $I$ of order $p$ be

$$I^T = [1, 1, ..., 1]$$ (29)

in which $p$ is the number of sensors. Now compute the following quantities for the $j$th mode shape.

$$\bar{c}_j = c_j^T I/p$$ (30)

$$c_r = \frac{\text{Re}(c_j - \bar{c}_j I)^T \text{[Re}(c_j - \bar{c}_j I)]}{p}$$ (31)

$$c_i = \frac{\text{Re}(c_j - \bar{c}_j I)^T \text{Im}(c_j - \bar{c}_j I)}{p}$$ (32)

$$c_u = \frac{\text{Im}(c_j - \bar{c}_j I)^T \text{Im}(c_j - \bar{c}_j I)}{p}$$ (33)

$$e = (c_u - c_r)/2c_i$$ (34)

$$\theta = \arctan \left[ e + \text{sgn}(e) (1 + e^2)^{1/2} \right]$$ (35)

where $\text{Re}( )$ and $\text{Im}( )$, respectively, are the real and imaginary parts of the complex vector $( )$, and $\text{sgn}( )$ is the sign of the scalar $( )$. The modal phase collinearity $\mu_j$ for the $j$th mode is then defined as

$$\mu_j = \frac{c_r + c_i [2(e^2 + 1)\sin^2(\theta) - 1]/\epsilon}{c_r + c_i}$$ (36)

This indicator checks the deviation from 0-180 degree behavior for components of the $j$th identified mode shape. The parameter $\mu_j$ can have only the values between 0 and 1. $\mu_j = 1$ indicates that the mode shape is high. On the other hand, if $\mu_j$ is away from 1, the $j$th mode is either a noise mode or the mode is significantly complex.

**Model Reduction**

The dynamical system is composed of an interconnection of all of the ERA-identified modes. The accuracy indicators allow one to determine the degree of individual mode participation. Model reduction can then be made by truncating all of the modes with low accuracy indicators. The accuracy of the complete modal decomposition process can be examined by comparing a reconstruction of $Y(k)$ formed by Eq. (16) with the original free decay responses, using the reduced model.

**Summary of ERA**

The computational steps are summarized as follows:

1) Construct a block-Hankel matrix $H_n(0)$ by arranging the measurement data into the blocks with given $r$, $s$, $t$, $(i = 1, 2, ..., s - 1)$ and $j, (i = 1, 2, ..., r - 1)$, [Eq. (4)].

2) Decompose $H_n(0)$ using singular-value decomposition [Eq. (10)].

3) Determine the order of the system by examining the singular values of the Hankel matrix $H_n(0)$ [Eq. (20)].

4) Construct a minimum-order realization $(A, B, C)$ using a shifted block-Hankel matrix [Eq. (16)].

5) Find the eigensolution of the realized state matrix [Eq. (22)] and compute the modal damping rates and frequencies [Eq. (23)].

6) Calculate the coherence parameter [Eq. (27)] and the collinearity parameter [Eq. (36)] to quantify the system and noise modes.

7) Determine the reduced system model based on the accuracy indicators, reconstruct function $Y(k)$ [Eq. (16)], and compare with the measurement data.

Note that the optimum determination of $r$, $s$, $t$, and $j$, in step 1 above requires further development. This determination is related to the choice of the measurement data to minimize the size of the Hankel matrix $H_n(0)$ with the rank unchanged.

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Fig. 1 Galileo spacecraft in launch configuration.
Example: Analysis of Galileo Test Data

The ERA method has been verified using multi-input and multi-output simulation data with or without noise for assumed structures with distinct and/or repeated eigenvalues. The reader is directed to the original version of this paper for more information. Experimental results for the analysis of Galileo test data are given in the following.

The Galileo spacecraft is shown in Fig. 1. All appendages, including the S-X-band antenna (SXA) at the top of the vehicle, were locked in their stowed positions. The structure was further secured to the test article with a massive seismic block. The adapter ring is the interface between Galileo and a Centaur upper stage that will provide the interplanetary boost.

Test and Data Acquisition Procedures

All results to be presented were obtained from two sets of free-response measurements recorded following single-point random excitation of the structure. The first data set was obtained using single-shaker, lateral excitation—in the global x direction—and the second set with single shaker, vertical excitation—in the global z direction. These tests are referred to as simply the "x direction" and "z direction" tests. For both tests, no special effort was made to select the position for the shaker. Each position was chosen using only the knowledge that many modes were excited from the location in previous tests. Other than the point and direction of excitation, all characteristics of the test, data acquisition, and data-reduction processes were exactly the same for both data sets.

The random excitation signal, bandlimited to the interval from 10 to 45 Hz, was generated digitally with the same test system which was also used to record the accelerometer response signals. Approximately 5 s of data were recorded following the end of the excitation signal. The responses were digitized at a rate of 102.4 samples per second, resulting in about 500 free-response points in each test.

Identified Eigenvalues

Each ERA analysis was performed using a single matrix of data from all 162 response measurements and one initial condition (either x- or z-direction test) at a time. Each response function Y as shown in Eq. (4) was thus a 162 \times 1 matrix. Using \( j_i = i \) and \( t_i = (i, 1, 2, \ldots, 499) \) the Hankel matrix \( H_k \) of 324 rows by 500 columns was formed to perform the analysis.

A summary of the identification results for the x direction test is provided in Table 1, including identified frequencies, damping factors, and accuracy indicators. The results for the z-direction test can be found in Ref. 24. These results closely agree with those obtained by other experimental techniques as shown in Refs. 17 and 25.

Table 1 ERA-identified results for x-direction

<table>
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<th>Mode No.</th>
<th>Frequency, Hz</th>
<th>Damping factor, %</th>
<th>Accuracy indicators</th>
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<td>98.6</td>
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<tr>
<td>2</td>
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\( a \) Modal amplitude coherence.
\( b \) Modal phase collinearity.
\( c \) Maximum modal participation in g's among the 162 components identified.
\( d \) Number of mode shape components of 162 with initial response amplitude >0.01 g.

Fig. 2 Example ERA-identified mode shapes illustrating various degrees of phase angle scatter.
The best single accuracy indicator now available is the modal amplitude coherence. Its value is used in Table 1 to rate the identified modes at various degrees of accuracy. The rating scale is noted in the key beneath the table. A brief description of the coherence is given in each of the three columns of the table. The data in the last two columns are two additional indications of the strength of the modal response signals relative to the instrumentation noise floor. These indicators are computed using modal participation values in physical units, which are the products of the mode shapes and the initial modal amplitudes.

Identified Mode Shapes

Two typical ERA-identified mode shapes are shown in Fig. 2. Based on these results and observations from other data not shown, the following conclusions can be drawn.

1) The local behavior of many of the Galileo modes—emplified by the antenna mode shown in Fig. 2a in which only about five measurements show motion—makes it more difficult to identify all 162 components of the mode shapes accurately because of the low response levels.

2) A good measure of the effects of noise on the mode shape accuracy is often indicated by the amount of scatter in the identified modal phase angles from the ideal 0-180 deg normal mode behavior. Of course, true complex-mode behavior needs to be differentiated from identification scatter due to noise and nonlinearity. The best remedy is to compare the results for the same mode obtained in several different tests.

3) The parameter referred to as the modal phase collinearity can be used to measure how closely the modal phase angle results for each mode cluster near 0 and 180 deg. Calculated using principal component analysis, it indicates the extent to which the information in each complex-valued mode shape is representable as a real-valued vector. It ranges from a value of zero for no collinearity to 100% for perfect collinearity.

4) Based on studies with simulated data, the accuracy of mode shapes showing clustering of the identified phase angles near 0 and 180 deg, such as in Fig. 2, can generally be accepted with little questioning. However, those modes with significantly more phase angle scatter should not be used without further confirmation.

5) Most mode-shape components whose identified phase angles are displaced from the 0- and 180-deg lines are those with the smallest amplitudes. This characteristic is consistently observed in the result shown in Fig. 2b. Small modal amplitude results for these components, however, usually indicate accurately that the response amplitude is, in fact, very small. This information is all that can be expected from a measurement standpoint, and is all that is required in many instances.

Identified Modal Amplitudes

The ERA modal amplitude coherence indicates the purity of the individual modal amplitude time histories. For each identified eigenvalue, a modal amplitude time sequence is obtained for each initial condition. These data provide a direct indication of the strength with which the mode was identified in the analysis. For strongly identified modes, the modal amplitude history is a pure, exponentially decaying sinusoid of the corresponding frequency and damping, which decays smoothly over the entire analysis interval. For weakly identified modes, the modal amplitude history is distorted. In particular, the history is a sequence of noise for any eigenvalue not corresponding to a structural mode.

Typical examples of modal amplitude results from the Galileo analysis are shown in Fig. 3. These data were selected to illustrate the variation in the purity of the modal amplitude histories for a mode which was strongly excited is only one of the two ERA tests.

The best single accuracy indicator now available is the modal amplitude coherence. Its value is used in Table 1 to rate the identified modes at various degrees of accuracy. The rating scale is noted in the key beneath the table. A brief description of the information in each of the three columns on the far right of the table is also contained in the keys. The significance of modal phase collinearity will be discussed in the next subsection. The data in the last two columns are two additional indications of the strength of the modal response signals relative to the instrumentation noise floor. These indicators are computed using modal participation values in physical units, which are the products of the mode shapes and the initial modal amplitudes.

The identified modal amplitude from the antenna mode shown in Fig. 2a in which only about five measurements show motion—makes it more difficult to identify all 162 components of the mode shapes accurately because of the low response levels.

A good measure of the effects of noise on the mode shape accuracy is often indicated by the amount of scatter in the identified modal phase angles from the ideal 0-180 deg normal mode behavior. Of course, true complex-mode behavior needs to be differentiated from identification scatter due to noise and nonlinearity. The best remedy is to compare the results for the same mode obtained in several different tests.

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Concluding Remarks

An eigensystem realization algorithm is developed for modal parameter identification and model reduction for dynamical systems. Two developments are given in this paper. First, a new approach is developed to derive the basic formulation of minimum realization for dynamical systems. As byproducts of this approach, two alternative less powerful algorithms, identified as algorithms (A1) and (A2), are derived. A special case of (A1) is shown to be equivalent to an approach currently in use in the structural dynamics field. Second, accuracy indicators are developed to quantify the participation of system modes and noise modes in the realized system model. In other words, the degree of controllability and observability for each participating mode is determined. A model reduction then can be made for controller design.

Based on the results of other analyses of simulated data, the parameters referred to as modal amplitude coherence and modal phase linearity are good indicators of the identification accuracy that is achieved. Using these indicators, approximately 15 modes from the Galileo analysis are judged to be of high accuracy. Less than 10 min of CPU time on a CDC mainframe computer were required for the Galileo analysis. However, additional research is needed to correctly assess the degree of accuracy achieved by those modes showing significant identification scatter. The effects of structural nonlinearities on the results also need further attention.

Important features of the eigensystem realization algorithm are summarized as follows.

1) From the computational standpoint, the algorithm is attractive since only simple numerical operations are needed.
2) The computational procedure is numerically stable.
3) The structural dynamics requirements for modal parameter identification and the control design requirements for a reduced state space model are satisfied.
4) Data from more than one test can be used simultaneously to efficiently identify closely spaced eigenvalues.
5) Computation requirements are moderate.

Appendix

In view of Eqs. (4) through (6), the measurement function $Y(k+1)$ can be obtained through either of two other algorithms [Eqs. (A1) and (A2)]. The algorithm (A1) is

$$Y(k+1) = E_p^T H_p(k) E_m = E_p^T V A^k W W^T V E_m$$

$$= E_p^T [H_p(1) A^k] E_m$$

and the algorithm (A2) is

$$Y(k+1) = E_p^T H_p(k) E_m = E_p^T V W [H_p^T V A W]^k E_m$$

$$= E_p^T H_p(0) [H_p^T H_p(1)]^k E_m$$

Hence, by Eq. (3), $H_p(1) A^k$, $H_p(0) E_m$, $E_p^T$ or $H_p^T H_p(1)$, $E_p^T H_p(0)$ is a realization. The system [Eqs. (1) and (2)] with realization (A1) will be transformed into the following equation:

$$\ddot{x}(k+1) = H_p(1) H_p^T \dot{x}(k) + H_p(0) E_m u$$

$$y(k) = E_p^T \dot{x}(k)$$

where $x(k) = V H^T \dot{x}(k)$. Or, using Eq. (A2),

$$(k+1) = H^T H_p(1) \ddot{x}(k) + E_m u$$

$$y(k) = E_p^T \dot{x}(k)$$

These realizations are not of minimum order, since the dimension of $x$ is the number of either columns or rows of the matrix $H_p(0)$ which is larger than the order $n$ of the state matrix $A$ for multi-input and multi-output cases. Examination of Eqs. (A1), (A2), and (16) reveals that algorithms (A1) and (A2) are special cases of ERA. Algorithm (A1) is formulated by inserting the identity matrix (6) on the right-hand side of the state matrix $A$ as shown in Eq. (A1). On the other hand, algorithm (A2) is obtained by inserting the identity matrix (6) on the left-hand side of the state matrix $A$ as shown in Eq. (A2). However, ERA is formed by inserting the identity matrix (6) on both sides of the state matrix $A$ as shown in Eq. (16). Because of the different insertion, algorithms (A1) and (A2) do not minimize the order of the state transition matrix. Mathematically, if the singular-value decomposition technique is not included in the computational procedures [see Eqs. (8) and (9)], Eqs. (A1) and (A2) cannot be numerically implemented, unless a certain degree of artificial noise and/or system noise are present. Noises tend to make up the rank deficiency of the generalized Hankel matrix $H_p(0)$ for algorithms (A1) and (A2). Since the degree of noise contamination is generally unknown, algorithms (A1) and (A2) are not recommended.

References


