Investigations of aeroelastic response for a system with continuous structural nonlinearities

Todd O'Neil
Texas A & M Univ., College Station

Heather Gilliatt
Texas A & M Univ., College Station

Thomas W. Strganac
Texas A & M Univ., College Station

AIAA, ASME, ASCE, AHS, and ASC, 37th, Structures, Structural Dynamics and Materials Conference, Salt Lake City, UT, Apr. 18, 19, 1996

Measurements and predictions of aeroelastic response are examined for a wing section mounted to permit continuous nonlinear structural response. Nonlinear behavior is introduced using a unique model support system designed to permit prescribed nonlinear stiffness characteristics. The wing section is limited to pitch and plunge response. Tailored experiments establish nonlinear responses such as limit cycle oscillations, and these results are predicted with an analytical model. (Author)
INVESTIGATIONS OF AEROELASTIC RESPONSE FOR A
SYSTEM WITH CONTINUOUS STRUCTURAL NONLINEARITIES

Todd O'Neil*, Heather Gilliatt*, and Thomas W. Strganac†
Texas A&M University
College Station, Texas 77843-3141

Abstract

Measurements and predictions of aeroelastic response are examined for a wing section mounted to permit continuous nonlinear structural response. Nonlinear behavior is introduced using a unique model support system designed to permit prescribed nonlinear stiffness characteristics. The wing section is limited to pitch and plunge response. Tailored experiments establish nonlinear responses such as limit cycle oscillations, and these results are predicted with an analytical model.

Nomenclature

\[
\begin{align*}
V_\infty &= \text{freestream velocity} \\
y &= \text{coordinate for plunge dof} \\
\alpha &= \text{coordinate for pitch dof} \\
\alpha_\infty &= \text{nondimensional system constant for pitch dof} \\
&\quad\quad\quad\quad\quad\quad\quad\quad\quad(= m_\infty g r / k_\alpha) \\
\beta &= \text{nondimensional nonlinear stiffness parameter for pitch dof} \\
y &= \text{nondimensional nonlinear stiffness parameter for plunge dof} \\
\delta &= \text{nondimensional stiffness parameter for pitch dof} \\
\zeta_\alpha &= \text{damping ratio for pitch dof due to viscous damping} \\
&\quad\quad\quad\quad\quad\quad(= c_\alpha / 2 m_\infty \omega_\alpha) \\
\zeta_\gamma &= \text{damping ratio for plunge dof due to viscous damping} \\
&\quad\quad\quad\quad\quad\quad(= c_\gamma / 2 m_\infty \omega_\gamma) \\
\theta &= \text{static angle of attack} \\
\mu &= \text{wing to air mass ratio} \\
&\quad\quad\quad\quad\quad\quad(= m_\infty / \rho b^2 S) \\
\mu_m &= \text{wing to system mass ratio} \\
&\quad\quad\quad\quad\quad\quad(= m_\infty / m_T) \\
\mu_\alpha &= \text{Coulomb damping coefficient for pitch dof} \\
\mu_\gamma &= \text{Coulomb damping coefficient for plunge dof} \\
\xi &= \text{nondimensional coordinate for plunge dof} \\
\xi_\alpha &= \text{nondimensional coordinate for pitch dof} \\
\xi_\infty &= \text{nondimensional system constant for plunge dof} \\
&\quad\quad\quad\quad\quad\quad(= m_\infty g / \rho k_\gamma) \\
\rho &= \text{density of air} \\
\sigma &= \text{nondimensional nonlinear stiffness parameter for plunge dof} \\
\tau &= \text{nondimensional time} \\
\phi &= \text{Wagner's function} \\
\omega &= \text{frequency ratio} \\
&\quad\quad\quad\quad\quad\quad(= \omega_\alpha / \omega_\gamma) \\
\omega_\alpha &= \text{frequency associated with the single dof motion in plunge} \\
\omega_\gamma &= \text{frequency associated with the single dof motion in pitch}
\end{align*}
\]

Introduction

Aeroelasticity is the dynamic interaction of structural, inertial, and aerodynamic forces. Conventional methods of examining aeroelastic behavior have relied on a linear approximation of the governing equations which describe the flowfield and the structure. However, aerospace systems inherently contain structural and aerodynamic nonlinearities (see Dowell et al.\(^1\), Dowell\(^2\), Jones and Lee\(^3\)). Dowell and...
Ilgamov\textsuperscript{4} attribute the success of linear flutter analysis to negligible nonlinear effects but showed that nonlinear effects are critical for many circumstances. Nonlinearities of significance result from unsteady aerodynamic sources, large deflections, and partial loss of structural or control integrity. Other mechanisms include large oscillations where flow separation may occur and structural damping mechanisms such as hydraulic and dry friction. These aeroelastic systems may exhibit nonlinear dynamic response characteristics such as limit cycle oscillations (LCOs), internal resonances, and chaotic motion.

Woolston et al.\textsuperscript{5} investigated the effect of freeplay, hysteresis, and cubic structural nonlinearities on flutter. Their results showed that the flutter instability was dependent upon the initial conditions for freeplay and hysteresis nonlinearities and showed that an increase in the initial displacements reduced the flutter velocity. The instabilities exhibited limit cycle oscillations which transition into divergent flutter regimes. Cubic hardening effects resulted in strictly limit cycle oscillations and the stability boundary was insensitive to initial conditions. In agreement, Scanlan and Rosenbaum\textsuperscript{6} recognized that it would be possible to allow the amplitude of the flutter oscillations to grow to a limiting amplitude through the introduction of nonlinear structural stiffness.

Analyses of freeplay and preload nonlinearities have been undertaken by several investigators (see Hauenstein et al.\textsuperscript{7}, Lee and Desrochers\textsuperscript{8}, Price et al.\textsuperscript{9,10}, Tang and Dowell\textsuperscript{11}, and Yang and Zhao\textsuperscript{12}). Tang and Dowell\textsuperscript{11} analyzed freeplay structural nonlinearities in the pitch mode using both linear and nonlinear aerodynamic models and showed that limit cycle oscillations depend upon freestream velocity, initial conditions, freeplay, and initial pitch angle. An interesting finding was that regions of limit cycle behavior occur for conditions below the linear flutter speed. These findings were also evident in the analyses by Lee and Desrochers\textsuperscript{4} and Price, et al.\textsuperscript{9}. Experimental investigations with piecewise linear structural response (Yang and Zhao\textsuperscript{12}) showed increased limit cycle amplitude for increased freestream velocity until the linear flutter velocity was reached at which point flutter occurred. Their results show two limit cycle amplitudes occurred at one velocity and show motion oscillated about a point other than the static equilibrium point.

The existence of chaotic motion for cases of structural preload has been analytically determined by Price et al.\textsuperscript{10} and Tang and Dowell\textsuperscript{11} and experimentally investigated by Hauenstein et al.\textsuperscript{7} and Tang and Dowell\textsuperscript{11}. For systems with freeplay nonlinearities, the results of these authors indicated transitions from limit-cycle oscillations to chaotic responses occur when the motion approached either the flutter boundary or the stall flutter boundary.

The effort described herein implements a model with continuous nonlinear structural stiffness and represents a progression of the recent research in piecewise linear models. Continuous nonlinear structural stiffness effects such as those exhibited by the response of a thin wing, rotor, or propeller subjected to large torsional amplitudes have been addressed by several researchers (see Dowell et al.\textsuperscript{11}, Woolston, et al.\textsuperscript{5}, Lee and LeBlanc\textsuperscript{13}, Price, et al.\textsuperscript{9,10}, Jones and Lee\textsuperscript{13}, and Zhao and Yang\textsuperscript{14}) but only Tang and Dowell\textsuperscript{11} have reported experimental studies of continuous nonlinear stiffness similar to those experiments described herein. The work of Lee and LeBlanc\textsuperscript{13} provides trends which relate the nonlinear response to variations in the initial conditions, nonlinear parameters, and physical system parameters. Lee and LeBlanc also showed that spring hardening effects experience only limit-cycle behavior, these cases did not experience divergent flutter. As the freestream velocity was increased, the amplitude of the limit-cycle oscillation increased and less time was required to reach the limit-cycle motion. The stability boundary of the limit cycle motion was insensitive to initial conditions. Chaotic response was predicted by Price et al.\textsuperscript{10} and Zhao and Yang\textsuperscript{14} for continuous hardening stiffness in incompressible flow. They determined that chaos occurred for certain elastic axis positions when the velocity was greater than the linear flutter speed.

Tang and Dowell\textsuperscript{11} compared analytical predictions to experimental measurements for continuous nonlinear stiffening behavior in the pitch mode. Most importantly, the existence of limit cycle behavior was found for quadratic and cubic pitching nonlinearities. These nonlinearities led to limit cycle oscillations which were dependent upon velocity and the nonlinear parameters. It was shown for continuous nonlinearities that the limit cycle motion consisted of many higher harmonic components in addition to the dominant flutter frequency. Their numerical and experimental results provide the best comparisons for the results of the investigations described herein.
Theory

The aeroelastic system is modeled as a wing limited to motion in two degrees of freedom. As illustrated in Fig. 1, a rigid wing section is mounted to a support system which permits plunge and pitch motion.

Fig. 1 The aeroelastic paradigm is a 2 dof system with nonlinear structural response and both Coulomb- and viscous-type damping.

The equations of motion are

\[ m\ddot{y} + \mu_r \cos(\alpha + \theta) \dot{\alpha} - \mu_r \sin(\alpha + \theta) \dot{\alpha}^2 + c_y \dot{y} + k_y \left( \sum_{n=1}^{\infty} n \dot{\alpha} \dot{y}^n \right) = -L \]  

and

\[ I_e \ddot{\alpha} + \mu_r \cos(\alpha + \theta) \dot{\alpha} + k_\alpha \left( \sum_{n=1}^{\infty} n \dot{\alpha} \right) = M \]  

where it is noted that the summations model a nonlinear structural stiffness, kinematic nonlinearities are retained in the equations of motion, and the aerodynamic loads, \( L \) and \( M \), are referenced to the quarter chord. As one should expect, the linear form of these equations agree with those widely published in the literature (see Fung).

In order to compare the model with results obtained with the experimental apparatus, it is necessary to consider the ratio of the mass of the wing to the total mass of the moving support structure which is not exposed to the flow (illustrated in Fig. 2). This mass ratio provides inertial coupling unique to the experimental apparatus and the necessity of this ratio will be further discussed in the Experiment section. In addition, Coulomb damping is introduced. This contribution manifests itself as a term which is dependent upon the velocity. Thus, the equations of motion are modified and are written in nondimensional form as

\[ \ddot{\xi} + \mu_r x_{\alpha} \cos(\alpha + \theta) \alpha'' - \mu_r x_{\alpha} \sin(\alpha + \theta) \alpha' \]  

\[ + 2\zeta_\alpha \left( \ddot{\xi}^2 + \dot{\xi}^2 \right) + \sum_{n=1}^{\infty} \sigma_n \xi^n \]  

\[ = p(\tau) - \mu_l \xi \left( \ddot{\xi} + \frac{k_\alpha}{m} \xi \right) \]  

and

\[ \alpha'' + \frac{\mu_r x_{\alpha}}{\tau_a} \cos(\alpha + \theta) \xi'' + 2\zeta_\alpha \left( \frac{1}{U} \right) \alpha' \]  

\[ + \left( \frac{1}{U} \right)^2 \sum_{n=1}^{\infty} \beta_n \alpha^n = r(\tau) - \mu_\alpha \frac{\alpha''}{U^2} \]  

where the prime denotes derivatives with respect to nondimensional time, \( \tau \), and Coulomb damping is represented by the last term in Eqs. (3) and (4). The nondimensional lift and moment are given as (see Lee and Leblanc)

\[ p(\tau) = -\frac{1}{U} (\xi'' - a\alpha'' + \alpha') - \frac{2}{U} \left[ (\alpha_o + \xi_o + \frac{1}{2} - a)\alpha_o' \right] \phi(\tau) \]  

\[ + \int_0^\tau \phi(\tau - \sigma)(\alpha'(\sigma) + \xi''(\sigma) + \frac{1}{2} - a)\alpha''(\sigma) d\sigma \]  

and

\[ r(\tau) = -\frac{2}{\mu_\alpha^2} \left( \frac{1}{2} + a \right) \left( \alpha_o + \xi_o + \frac{1}{2} - a \right) \alpha_o' \phi(\tau) \]  

\[ + \int_0^\tau \phi(\tau - \sigma)(\alpha'(\sigma) + \xi''(\sigma) + \frac{1}{2} - a)\alpha''(\sigma) d\sigma \]  

\[ + \frac{1}{\mu_\alpha^2} (a\xi'' - \frac{1}{2} - a)\alpha' - \left( \frac{1}{2} + a^2 \right) \alpha'' \]  

where \( \phi \) is Wagner's function (see Fung) which gives the growth of circulation about the wing due to a step input. An approximation for Wagner's function is given by Jones as

\[ \phi(\tau) = 1 - 0.165e^{-0.045\tau} - 0.335e^{-0.3\tau} \]  

The lift and moment are dependent upon the current acceleration, velocity, and position as well as the history of motion and the initial conditions. This aerodynamic model allows an arbitrary history of airfoil motion to be considered but does not account for flow nonlinearities such as stall. The convolution...
integral in Eqs (5) and (6) must be evaluated at each time step but a recurrence formula for the integral is employed (Lee and Desrochers) to improve efficiency. The formulae and starting values for the recursion method are presented in O’Neil.

The aeroelastic response is simulated by integration based on the model introduced by Lee and LeBlanc and Lee and Desrochers. The finite difference scheme of Houbolt is used (also see O’Neil). The velocity and acceleration are determined using the IMSL nonlinear solution package (DNEQNF). The initial conditions for the solution of these equations of motion include only the initial displacements in plunge and pitch. The initial velocities are zero which match the measured initial conditions for the experiment. The analysis is limited to third-order accuracy. The integration time step is 1/256 of the shorter period of the two free vibration modes (i.e., in the absence of aerodynamic loads). The time step size was determined by Lee and LeBlanc to give good accuracy in predicting the flutter boundaries.

**Experiment**

A model support system (see the illustration in Fig. 2) has been developed to provide experimental investigations of nonlinear aeroelastic response. The support system permits pitch and plunge motion for a mounted model. For the present studies, a rigid wing section is mounted to the support structure and provides bending- and torsion-type behavior of a wing. The plunge degree of freedom is provided by a carriage which slides on translational bearings. Two rotational bearings are mounted on this carriage and provides pitch motion independent of plunge motion. The pitch and plunge response are prescribed by the selection of springs. Protective constraints limit the amplitude of motion to prevent damage from flutter. These constraints are located well outside the range of motion required for flutter to be present. Typically, only the mass of the wing is considered in analysis; however, since the carriage moves with the attached wing, the mass of both the wing and support structure are used in the analysis. In addition, the moment of inertia associated with the pitch motion does not reflect the total mass since a portion of the system does not pitch.

The model support system provides freedom in test conditions and parameters. The structural stiffness response of the apparatus is governed by a pair of cams designed to provide tailored linear or nonlinear stiffness response. The shape of each cam, stiffness of the springs, and pretension in the springs dictates the nature of the nonlinearity. With this approach, these cams provide continuous nonlinear stiffnesses in pitch and plunge degrees of freedom and, in addition, a large family of prescribed possibilities may be investigated. Other physical properties -- such as the eccentricity of the aerodynamic center, mass of system components, the moment of inertia of the wing, stiffness characteristics, and the wing shape -- are easily modified for parametric investigations.

![Fig. 2](image-url)  
*Fig. 2 The experimental test apparatus is illustrated -- cams of prescribed shape dictate the nonlinear response.*
An accurate model of the structural damping is important. Most aeroelastic models described in the literature treat the structural damping as a viscous damping term; however, due to the design of this experiment, damping forces cannot be solely attributed to viscous damping effects. The bearings in the experiment generate a rolling friction which is best modeled by Coulomb damping (Kelly\textsuperscript{21}); thus, the additional terms found in Eqns. (3) and (4) are necessary. Scanlan and Rosenbaum\textsuperscript{6} suggest that, for those cases in which damping forces are modeled with Coulomb friction, the damping coefficients are nonlinear since there may be significant damping loads for small velocities yet small damping loads for large velocities. Dowell and Ilgamov\textsuperscript{4} stated that dry friction damping is one of the important nonlinear effects that resists rational models.

System response is measured with accelerometers and optical encoders mounted to track motion in each degree of freedom. Experiments are conducted by setting a freestream velocity and initial conditions, releasing the structure, and measuring the response. Initial conditions are set by dictating both a pitch and plunge initial condition, or dictating a pitch condition and allowing the plunge response to displace freely. The initial conditions are displacements -- no initial velocities are used. The accelerations are acquired using an analog to digital board, and the frequency content of the measurements are analyzed using Fast Fourier Transform (FFT) techniques. The optical encoders require digital input/output boards and output storage. Measurements are examined to determine frequency and stability characteristics prior to increasing the freestream velocity to the next test point. Flutter is determined by monitoring the frequency and damping of the measured motion as well as observation of the system response.

**Results**

Physical properties of the wing and model support system are presented in Table 1. The nonlinear stiffness of the model support system is modeled using a polynomial fit of the measured response (Fig. 3). The stiffness behavior for the pitch degree of freedom is

\[
 k_\alpha = (\sum_{n=1}^{\infty} y_n \alpha^n) = 2.820 \, (1.0\alpha - 22.1\alpha^2 + 1316.0\alpha^3 - 8580.0\alpha^4 + 17290.0\alpha^5) \, \text{N-m/rad}
\]

for the nonlinear system, and for these present investigations, the stiffness behavior for the plunge degree of freedom is constant.

<table>
<thead>
<tr>
<th>parameter</th>
<th>value</th>
</tr>
</thead>
<tbody>
<tr>
<td>a</td>
<td>-0.8</td>
</tr>
<tr>
<td>m_T</td>
<td>12.0 kg</td>
</tr>
<tr>
<td>b</td>
<td>0.1064 m</td>
</tr>
<tr>
<td>k_w</td>
<td>3.430 N-m/rad</td>
</tr>
<tr>
<td>k_r</td>
<td>2844.0 N/m</td>
</tr>
<tr>
<td>I_r</td>
<td>0.0505 kg-m\textsuperscript{2}</td>
</tr>
<tr>
<td>S</td>
<td>0.6 m</td>
</tr>
<tr>
<td>m_w</td>
<td>2.38 kg</td>
</tr>
<tr>
<td>r</td>
<td>0.084 m</td>
</tr>
<tr>
<td>\mu</td>
<td>459.0</td>
</tr>
<tr>
<td>\mu_n</td>
<td>0.0125</td>
</tr>
<tr>
<td>\mu_w</td>
<td>0.0252</td>
</tr>
<tr>
<td>M_f</td>
<td>0.878 N-m</td>
</tr>
</tbody>
</table>

Experiments with the linear support structure indicate that flutter due to stall occurs for pitch amplitudes approaching 0.25 radians. LCO behavior is measured for the nonlinear stiffness case. The nonlinear cam results in LCOs with an amplitude significantly below the stall flutter amplitude. Figure 4 shows the measured response of the system from its release at a pitch initial condition of 0.174 radians and a freestream velocity of 15.2 m/s. Initially, the system appears to have damped behavior; however, after the transients

![Fig. 3 The shape and associated response of the nonlinear cam is represented as a polynomial.](image-url)
disappear, aerodynamic feedback develops and leads to LCO behavior. The system frequencies coalesce at 2.76 Hz. The nonlinear stiffness captures the oscillations for pitch at amplitudes of 0.12 radians and for plunge at amplitudes of 0.0032 m. Limit cycle oscillations are observed over a velocity range of 10 m/s with little increase in oscillation amplitude.

The influence of initial conditions and structural damping on the stability boundary is examined. Figure 5 maps the measured and predicted response of the nonlinear system for several initial conditions. Regions of stability and LCO behavior are shown. Predictions by Woolston and Lee and LeBlanc showed that the stability boundary for systems with cubic hardening stiffness is insensitive to initial conditions for models using viscous damping. Predictions with the model presented herein, but limited to viscous damping only, provide a stability boundary which is also insensitive to initial conditions. Yet, as shown in Fig. 5, predictions which include Coulomb-type damping indicate a stability boundary which is sensitive to initial conditions. Furthermore, these predictions are confirmed by experimental measurements as indicated in Fig. 5.

The measurements obtained from the experiments agree with the predictions from the analytical model as well as the predictions of other researchers as discussed in the Introduction. Tang and Dowell provide the only source of comparison for measured LCO behavior obtained from experiments with continuous nonlinearities. Their work showed comparable LCO behavior -- both theory and experiments -- and their efforts indicate that oscillations for pitch at amplitudes of approximately 0.10 radians can be predicted successfully using linear aerodynamic theory. Comparisons with the measured amplitudes presented herein (0.12 radians) provides evidence that the experiment is indeed experiencing LCO behavior due to structural nonlinearities and that analysis using linear aerodynamic theory is appropriate.

The influence of freestream velocity on the amplitude for the pitch LCO, amplitude for the plunge LCO, and frequency of oscillation is presented in Figs. 6, 7, and 8, respectively. Measured and predicted results are shown. Several initial conditions were used for each freestream velocity, and in these figures each point actually represents the results of several initial conditions (pitch displacement). Thus, the figures confirm that for a given freestream velocity, the LCO will have the same amplitude and frequency regardless of the initial condition, and the results further suggest that if conditions are present for LCO behavior then the motion is insensitive to pitch initial conditions but dominated by freestream velocity. It is observed in Figs. 6 and 7 that the amplitude for the LCO grows with an increase in freestream velocity as one might expect. This trait is evident for both measured and predicted results. Measurements of the LCO frequency

Fig. 4 Measurements of nonlinear response are shown for $V_\infty = 15.2$ m/s.

Fig. 5 The measured and predicted stability boundary is found for several initial conditions.
for increasing freestream velocities show negligible change (Fig. 8); whereas, predictions of the LCO frequencies show a noticeable increase as the freestream velocity is increased. Similar trends have been measured and predicted by Tang and Dowell\textsuperscript{15} as well as the predictions of Lee and LeBlanc\textsuperscript{13}.

Predicted and measured time histories of motion show that the motion does not oscillate about the static equilibrium position (Fig. 9). Rather, the mean value for the LCO oscillation is shifted for both the pitch and plunge modes. This behavior was also noted by Yang and Zhao\textsuperscript{12} for cases with freeplay nonlinearities. The cause for this shift from the origin is attributed to the nature of the nonlinearity (see Nayfeh and Mook\textsuperscript{22}). Comparisons of the measured and predicted shift show good agreement.

Fast Fourier Transform (FFT) methods are used to examine the frequency content of the response. Figure 10 compares FFTs derived from predictions and measurements of the system response. The FFTs exhibit higher harmonics, especially in the plunge degree of freedom, caused by the structural nonlinearity of the system. The predicted frequency of the fundamental mode is higher than the measured frequency. Tang and Dowell\textsuperscript{15} showed comparable FFTs from experiment and analysis.

**Conclusions**

The authors examine experimental and analytical nonlinear aeroelastic behavior. A unique model support apparatus is described which has been developed to permit investigations of tailored nonlinear aeroelastic response. The apparatus allows plunge and pitch motion controlled by a set of cams and springs which prescribe linear or nonlinear stiffness response. An analytical model is presented and the predictions are verified by the efforts of several other investigators,
such as the work of Lee and LeBlanc, Lee and Desrochers, and Tang and Dowell. The analytical model uses Houbolt's finite difference method for describing velocities and accelerations. The analysis considers both viscous- and Coulomb-type damping.

The frequency content for LCO episodes show higher harmonics. The frequency content of the limit cycle response exhibits higher harmonics. Comparisons with limit cycle behavior found by the experimental investigations of Tang and Dowell and the analytical studies of Lee and LeBlanc show good agreement with this work.

Acknowledgment

This research is partially supported by Grant CMS-9502567 from the National Science Foundation. The authors are grateful for this support.

References


