Work Done by Force Acting on a Uniaxial Bar

Let’s assume
- length of bar = 200 inches
- x-sectional area of bar = .05 square inches
- density of material = .1 lbm/cubic inch
- load P increases from 0 to 100 lbs in 10 sec (see graph)
- movement at the point where load is applied varies linearly with the applied load
- when the load is 100 lbs, the displacement at the right end = 1 inch
- the displacement is zero at the left end and varies linearly from one end to the other

- What is the velocity at the right end during the 10 seconds
- What is $v(x)$?
- How much work is done by the force?

Now let’s see what COE has to say.

$$\frac{\partial (\dot{U})}{\partial t} \rho = -\frac{\partial \dot{U}}{\partial x} \rho v_x + \sigma_{xx} \frac{\partial v_x}{\partial x} - \frac{\partial q_x}{\partial x} + \rho \Phi$$

There is no heat source and let’s assume there is negligible heat conduction.
Also, it is reasonable to assume that $\dot{U}$ is constant along the length of the bar =>

$$\frac{\partial (\dot{U})}{\partial t} \rho = \sigma_{xx} \frac{\partial v_x}{\partial x}$$ since $\dot{U}$ is only a function of time and $v_x$ is only a function of “x”, this can be written as

$$\frac{d(\dot{U})}{d t} \rho = \sigma_{xx} \frac{dv_x}{dx}$$

or differential change in internal energy = $volume \rho d(\dot{U}) = volume \sigma_{xx} \frac{dv_x}{dx} dt$

Integrate to obtain the change

The change in internal energy in the bar is

$$\int_{final}^{initial} volume \rho d(\dot{U}) = \int_0^{10} volume \sigma_{xx} \frac{dv_x}{dx} dt$$

Compare this answer with the earlier answer for work done by the force “P”.