4.34

Boundary Layer

\[ \text{N} = 1.749 \times 10^{-5} \, \text{m}^3 \]

\[ V = 200 \, \text{m/s} \]

\[ \text{Re} = 1.25 \times \text{m} \]

\[ \text{S} = 52.5 \]

\[ C_x = \frac{5.3 \times 10^2}{\text{Re}^{1/2}} \]

\[ \text{Re} = \frac{V \, \rho \, S \, C_x}{\mu} \]

\[ \text{Re}_k = 4.107 \times 10^9 \]

\[ \delta_3 = 0.0034 \, \text{m} \]

Total Skin Friction Drag

\[ \text{Re}_L = \text{Re}_k \]

\[ C_f = \frac{1.324 \times 10^{-4}}{\text{Re}_L^{1/4}} = 0.00201 \]

\[ D_c = \frac{1}{2} \rho V^2 S C_f \]

\[ D_c = \frac{1}{2} (1.225) (200)^3 (52.5) (0.00201) = 2.662 \, \text{N} \]

\[ D_{\text{car}} = 2 D_c = 5325 \, \text{N} \]

Boundary Layers

\[ \delta = \frac{\text{Re}_x}{\text{Re}_k} = \frac{3.7 (3.5)}{(4.11 \times 10^9)^{1/2}} = 0.0333 \, \text{m} \]

4.35

Total Skin Friction Drag

\[ C_f = \frac{0.074}{\text{Re}_L} = 0.00207 \]

\[ D_c = \frac{1}{2} \rho V^2 S C_f \]

\[ D_c = \frac{1}{2} (1.225) (200)^3 (52.5) (0.00207) = 2.69 \, \text{N} \]

\[ D_{\text{car}} = 2 D_c = 5660 \, \text{N} \]

\[ \frac{D_{\text{car}}}{D_{\text{car}}} = \frac{0.00243}{0.0333} = 0.072 \approx \frac{1}{13.7} \]

\[ \frac{D_{\text{car}}}{D_{\text{car}}} = \frac{5325}{5660} = 0.94 \approx \frac{1}{10.6} \]
4.36 \[ Re_{ce} = \frac{\rho_{\infty} V_{\infty} x_{ce}}{\mu_{\infty}} \]

\[ x_{ce} = Re_{ce} \left( \frac{\mu_{\infty}}{\rho_{\infty} V_{\infty}} \right) = \frac{(10^6)(1.789 \times 10^{-5})}{(1.225)(200)} \]

\[ x_{ce} = 7.3 \times 10^{-2} \text{m} \]

\[ \Downarrow \quad V_{\infty} \approx 200 \text{ m/sec} \]

\[ \Downarrow \quad 7.3 \times 10^{-2} \text{m} \]

\[ \text{LAMINAR FLOW} \quad A \]

\[ \text{TURBULENT FLOW} \quad B \]

The turbulent drag that would exist over the first \( 7.3 \times 10^{-2} \text{ m} \) of chord length from the leading edge (area \( A \)) is

\[ D_{fa} = \frac{0.074}{(Re_{ce})^{0.2}} q_{\infty} S_A \] (on one side)

\[ D_{fa} = \frac{0.074}{(10^6)^{0.2}} (2.45 \times 10^4)(7.3 \times 10^{-2})(17.5) \]

\[ D_{fa} = 146 \text{ N} \quad \text{(on one side)} \]

From Problem 4.25, the turbulent drag on one side, assuming both areas \( A \) and \( B \) to be turbulent, is 2830N. Hence, the turbulent drag on area \( B \) alone is:

\[ D_{fb} = 2830 - 146 - 2684 \text{ N} \quad \text{(turbulent)} \]

The laminar drag on area \( A \) is

\[ D_{fa} = \frac{1.328}{(Re_{ce})^{0.5}} q_{\infty} S_a \]

\[ D_{fa} = \frac{1.328}{(10^6)^{0.5}} (2.45 \times 10^4)(7.3 \times 10^{-2})(17.5) \]

\[ D_{fa} = 42 \text{ N} \quad \text{(laminar)} \]

Hence, the skin friction drag on one side, assuming area \( A \) to be laminar and area \( B \) to be turbulent is

\[ D_f = D_{fa} \text{ (laminar)} + D_{fb} \text{ (turbulent)} \]

\[ D_f = 42 + 2684 = 2726 \text{ N} \]

The total drag, accounting for both sides, is

\[ \text{Total } D_f = 5452 \text{ N} \]

*Note*: By comparing the results of this problem with those of Problem 4.25, we see that the flow over the wing is mostly turbulent, which is usually the case for real airplanes in flight.
5.2 From Appendix D, at 5° angle of attack,

\[ c_l = 0.67 \]

\[ c_{m_4} = -0.025 \]

(Note: Two sets of lift and moment coefficient data are given for the NACA 1412 airfoil -- with and without flap deflection. Make certain to read the code properly, and use only the unflapped data, as given above. Also, note that the scale for \( c_{m_4} \) is different than that for \( c_l \) -- be careful in reading the data.)

With regard to \( c_l \), first check the Reynolds number,

\[ Re = \frac{\rho \cdot V \cdot c}{\mu} = \frac{(0.002377)(100)(3)}{3.7373 \times 10^{-7}} \]
Re = 1.9 x 10^6

In the airfoil data, the closest Re is 3 x 10^6. Use \( c_d \) for this value.

\[ c_d = 0.007 \quad (\text{for} \quad c_f = 0.67) \]

The dynamic pressure is

\[ q_\infty = \frac{1}{2} \rho_\infty V_\infty^2 = \frac{1}{2} (0.002377)(100)^2 = 11.9 \text{ lb/ft}^2 \]

The area per unit span is \( S = 1(c) = (1)(3) = 3 \text{ ft}^2 \)

Hence, per unit span,

\[ L = q_\infty S \ c_f = (11.9)(3)(0.67) = 23.9 \text{ lb} \]

\[ D = q_\infty S \ c_d = (11.9)(3)(0.007) = 0.25 \text{ lb} \]

\[ M_{\mu u} = q_\infty S \ c \ c_{\mu u} = (11.9)(3)(3)(-0.025) = -2.68 \text{ ft.lb} \]

\[ \rho_\infty = \frac{p_\infty}{RT} = \frac{(1.01 \times 10^5)}{(287)(303)} = 1.61 \text{ kg/m}^3 \]

From Appendix D,

\[ c_f = 0.98 \]

\[ c_{\mu u} = -0.012 \]

Checking the Reynolds number, using the viscosity coefficient from the curve given in Chapter 4,

\[ \mu_\infty = 1.82 \times 10^{-5} \text{ kg/m sec at } T = 303 \text{K}, \]

\[ \text{Re} = \frac{\rho_\infty V_\infty c}{\mu_\infty} = \frac{(1.157)(42)(0.3)}{1.82 \times 10^{-5}} = 8 \times 10^5 \]
This Reynolds number is considerably less than the lowest value of $3 \times 10^6$ for which data is given for the NACA 23012 airfoil in Appendix D. Hence, we can use this data only to give an educated guess; use

$c_d = 0.01$, which is about 10 percent higher than the value of 0.009 given for $Re = 3 \times 10^6$

The dynamic pressure is

$$q_\infty = \frac{1}{2} \rho_\infty V_\infty^2 = \frac{1}{2} (1.161)(42)^2 = 1024 \text{ N/m}^2$$

The area per unit span is $S = (1)(0.3) = 0.3 \text{ m}^2$. Hence,

$$L = q_\infty S \, c_r = (1024)(0.3)(0.98) = 301 \text{ N}$$

$$D = q_\infty S \, c_d = (1024)(0.3)(0.01) = 3.07 \text{ N}$$

$$M_{cd} = q_\infty S \, c_m = (1024)(0.3)(0.3)(-0.012) = -1.1 \text{ Nm}$$

5.4 From the previous problem, $q_\infty = 1020 \text{ N/m}^2$

$$L = q_\infty S \, c_r$$

Hence,

$$c_r = \frac{L}{q_\infty S}$$

The wing area $S = (2)(0.3) = 0.6 \text{ m}^2$

Hence,

$$c_r = \frac{200}{(1024)(0.6)} = 0.33$$

From Appendix D, the angle of attack which corresponds to this lift coefficient is

$$\alpha = 2^\circ$$
5.5 From Appendix D, at \( \alpha = 4^\circ \),
\[
c_f = 0.4
\]
Also,
\[
V_\infty = 120 \left( \frac{88}{60} \right) = 176 \text{ ft/sec}
\]
\[
q_\infty = \frac{1}{2} \rho_\infty V_\infty^2 = \frac{1}{2} (0.002377)(176)^2 = 36.8 \text{ lb/ft}^2
\]
\[
L = q_\infty S \ c_f
\]
\[
S = \frac{L}{q_\infty c_f} = \frac{29.5}{(36.8)(0.4)} = 2 \text{ ft}^2
\]

5.6 \( L = q_\infty S \ c_f \)

\( D = q_\infty S \ c_d \)

Hence,
\[
\frac{L}{D} = \frac{q_\infty S \ c_f}{q_\infty S \ c_d} = \frac{c_f}{c_d}
\]

We must tabulate the values of \( c_f/c_d \) for various angles of attack, and find where the maximum occurs. For example, from Appendix D, at \( \alpha = 0^\circ \),
\[
c_f = 0.25
\]
\[
c_d = 0.006
\]

Hence
\[
\frac{L}{D} = \frac{c_f}{c_d} = \frac{0.25}{0.006} = 41.7
\]
A tabulation follows.

<table>
<thead>
<tr>
<th>$\alpha$</th>
<th>0°</th>
<th>1°</th>
<th>2°</th>
<th>3°</th>
<th>4°</th>
<th>5°</th>
<th>6°</th>
<th>7°</th>
<th>8°</th>
<th>9°</th>
</tr>
</thead>
<tbody>
<tr>
<td>$c_f$</td>
<td>0.25</td>
<td>0.35</td>
<td>0.45</td>
<td>0.55</td>
<td>0.65</td>
<td>0.75</td>
<td>0.85</td>
<td>0.95</td>
<td>1.05</td>
<td>1.15</td>
</tr>
<tr>
<td>$c_d$</td>
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<td>0.006</td>
<td>0.006</td>
<td>0.0065</td>
<td>0.0072</td>
<td>0.0075</td>
<td>0.008</td>
<td>0.0085</td>
<td>0.0095</td>
<td>0.0105</td>
</tr>
<tr>
<td>$c_f/c_d$</td>
<td>41.7</td>
<td>58.3</td>
<td>75</td>
<td>84.6</td>
<td>90.3</td>
<td>100</td>
<td>106</td>
<td>112</td>
<td>111</td>
<td>110</td>
</tr>
</tbody>
</table>

From the above tabulation,

\[
\frac{L}{D}_\text{max} = 112
\]

5.7 At sea level

$\rho_\infty = 1.225 \text{ kg/m}^3$

$p_\infty = 1.01 \times 10^5 \text{ N/m}^2$

Hence,

$q_\infty = \frac{1}{2} \rho_\infty V_\infty^2 = \frac{1}{2} (1.225)(50)^2 = 1531 \text{ N/m}^2$

From the definition of pressure coefficient,

$C_p = \frac{p - p_\infty}{q_\infty} = \frac{(0.95 - 1.01) \times 10^5}{1531} = -0.91$

5.8 The speed is low enough that incompressible flow can be assumed. From Bernoulli’s equation,
\[ p + \frac{1}{2} \rho V_w^2 = p_m + \frac{1}{2} \rho m V_m^2 = p_m + q_m \]

\[ C_p = \frac{p - p_m}{q_m} = \frac{q_m - \frac{1}{2} \rho V^2}{q_m} = 1 - \frac{\frac{1}{2} \rho V^2}{\frac{1}{2} \rho m V_m^2} \]

Since \( \rho = \rho_m \) (constant density)

\[ C_p = 1 - \left( \frac{V}{V_m} \right)^2 = 1 - \left( \frac{62}{55} \right)^2 = 1 - 1.27 = -0.27 \]

5.9 The flow is low speed, hence assumed to be incompressible. From problem 5.8,

\[ C_p = 1 - \left( \frac{V}{V_m} \right)^2 = 1 - \left( \frac{195}{160} \right)^2 = -0.485 \]