Trebuchet and Projectile Dynamics
Contributing members:

Anton Kolomiets

Russell Trahan

Zach Itkoe

Bryan Dickson
Table of Contents:

Introduction.................................................................................................................. 4
Problem Formulation........................................................................................................ 4-5
Procedures and Safety...................................................................................................... 5-7
Analytical calculations..................................................................................................... 7-8
Experimental Results, Comparison, Discussion.............................................................. 8-9
Conclusion........................................................................................................................ 9-10
Works Cited..................................................................................................................... 11
Appendices...................................................................................................................... 12-16
Introduction

For over a thousand years, people have used various techniques to move an object through the air. Whether for warfare or simple transportation, the ability to propel something over long distance has been vital to the growth and evolution of civilization. During the middle ages, before the advent of the cannon, the most common and popular way to shoot a projectile was the trebuchet. The main idea behind the trebuchet is that a large mass is used on one side of a long moment arm. The other side of the arm holds the projectile. A pivot point is positioned closer to the mass, allowing the entire arm assembly to rotate around it. The projectile itself is much less massive than the mass employed to move the arm and is held in place by a cup-type or structure. The design of the cup is important to the overall finished product – the cup serves not only to hold the projectile in place during initial acceleration, but also helps aim the projectile and helps it attain whichever flight specifics are required. When the trebuchet is ready to fire, the arm is held at a certain pre-determined angle – at this angle, the mass is at a higher position in the air (with a large potential energy), while the projectile is very low (almost 0 energy). The mass is then released and induces a moment about the arm. The projectile at the other end of the arm experiences this same moment and, since it is not connected to the arm in any way, is propelled through the air. Although a rather simplistic approach to projectile motion, one of the advantages to using a trebuchet is that the results of this motion are precise and very repeatable. The mass exerting a moment on the trebuchet’s throwing arm remains the same, and thus, if pulled to the same initial angle of launch, the mass would exert the same moment during each subsequent launch. As long as the trebuchet is aimed to the same location and all outside forces such as wind and air resistance are taken into account, the projectile should land in the same location each time it is launched.

Problem Formulation:

The overall goal for this project is to build a trebuchet and describe its motion, and the motion of its projectile, using the principles of dynamics. To build the trebuchet, we first formulated a set of rough dimensions and set out some goals for its motion (see appendix 1). We opted to build the machine out of wood due to its overall ease of use and inherent durability. Our initial plans called for a rectangular base, about 42 inches wide and 58 inches long. The actual vertical support structure for the throwing arm of the trebuchet is positioned 35.25 inches from the front of the base and only 20.5 inches ahead of the rear portion of the base. The logic behind placing the vertical supports of the throwing arm farther back on the base was that the at the end of its motion, the throwing arm would have to come to an abrupt stop to propel the projectile out of the support cup. The angular momentum of the throwing arm at the end of its motion would tend to tip the entire trebuchet forward – leaving outing the arm further back would allow for greater resistance to tipping the trebuchet. The base is also
designed to be a bit larger than the entire structure requires to allow for the use of sandbags or another sort of system to help anchor the machine to the ground. The vertical support structures were engineered to the precise height specifications required by a large, swinging 2x4 wooden arm. They are 40 inches tall each and are supported by angled supports along the length and width of the trebuchet. We chose to support each of the joints throughout the trebuchet with steel plates. Since the machine would have to survive many launches and relatively hard impacts from the throwing arm, the steel plates would make our machine much more durable and able to withstand the worst impacts. These plates are held in place with 1.75-inch-long screws along every joint except for the base joints (base joints were nailed together). The arm itself is a 96-inch-long 2x4. We positioned the pivot point 24 inches from the end of moment arm (area where the mass is located) making the action arm 72 inches. This 24 inch length of the moment arm allows plenty of space between the mass and the bottom of the base (40 inches due to the vertical support). We engineered the arm so that it could be pulled back 45 degrees below the horizontal and could fire to 45 degrees above the horizontal. This layout allows for a “mechanical advantage” of 1:3 from the mass that serves to create the moment about the pivot to the projectile at the end of the action arm.

Building the trebuchet required some precise calculations and the use of technology. We formulated a rough model of what we wanted to accomplish (appendix 1) with all of the required supports, pivot points, angles, joints, etc. We modeled our trebuchet in SolidWorks to make certain that all of our proposed cuts, angles, and joints would fit together into a working system. Our SolidWorks model turned out to be quite helpful with the actual calculation and building process associated with the trebuchet. It allowed us to change the dimensions of certain parts of the trebuchet that were causing problems and immediately see our new, reworked designs and their effectiveness within the system.

**Procedures and Safety Protocols**

When it came time to test our trebuchet, we took no chances. Before we ever launched our projectile for the first time, we had worked out the theoretical outcome. Based upon our calculations, we could determine how far we should expect our projectile to fly and could make arrangements to catch it or otherwise limit its motion after it initially hit the ground (see calculations to follow and appendix 2). The actual launching of our projectile was in itself an exercise in caution. We used 4 loops of 150-lb test fishing line to secure our throwing arm in the initial launch position. We wanted to be far enough away from it that, in the case of failure, the team would avoid any potential injury. To be able to disengage the fishing line while remaining relatively far away from the machine, we devised a “burn-out” trigger. The fishing line is connected to the bottom of the throwing arm via a mounting hook and then passes through another such hook on the base of the machine. From here, the line is held in place screwed securely to a circuit connector (also attached to the base).
To sever the line and launch the catapult, we attached a small, high-resistance wire to one of the adjacent circuit ports and wrap it tightly around the fishing line before attaching it to the second electrical port. We created a battery system and an electrical switch to supply power to the wire – this wire becomes extremely hot upon application of an electrical current and serves to cleanly and instantly cut the fishing line. This burn-out wire system allows the group to stand about 20 feet away from the trebuchet and launch it by remote control. Of course, we took further precautions to assure that our experiment would be as safe and as informative as it could be. The weights we used to launch the projectile were safety-wired together to prevent them from falling off of the connecting rod and injuring anyone. The projectile we employed was a simple baseball. Upon firing the projectile, we made sure that everyone and everything that could be damaged by a baseball was well out of its expected path of travel.
Our experimental procedures were quite straightforward and effective. We would first do the calculations to determine where our projectile should, theoretically, end up. Wind speeds and average gusts, if any were present at time of launch, were taken into account when completing the calculations. We made sure the trebuchet was standing on a relatively straight and level surface so as to minimize error in data collection due to less-than-perfect real-world conditions. Using our burn-out wire launch system, we were able to stand far enough away from the machine to see its complete motion and the full trajectory of our projectile. Typically, we would have one of our group members operating the launch switch behind the trebuchet, another one standing off to the side to watch the motion of the projectile, and another standing close to where we expect our projectile to make contact with the ground – this group member would then record the position of the ball’s final touchdown.

At the end of the run, we would measure the distance the ball flew from the trebuchet (projection of the throwing arm’s final position onto the ground) to the point of contact with the ground. The data we collected from our theoretical calculations and the data from our actual test shots is given in full and explained below.

**Analytical Calculations**

The key to predicting our projectiles trajectory and the distance it will travel is the velocity and angle at which the projectile departs from the throwing arm. Our approximation took into account three moments about the pivot point of the throwing arm. The first moment applied around the pivot point was due to the weight of the ball (treated as a point-mass). The second
moment was due to the weight of a section of the lever arm (we assumed that the density of the wood used on the lever arm was constant, thus parts of equal length on either side of the pivot would have equal and opposite moments that would cancel each other out). The third moment was due to the actual mass responsible for propelling the projectile – we used two different weights throughout this experiment to prove that our derived relationships and equations could be modified to account for any weight and still predict the distance our projectile could travel. The summation of these three torques divided by the moment of inertia gives the angular acceleration – alpha. The velocity with which the projectile leaves the throwing arm can then be calculated from the acceleration through a numerical integration. Once the velocity is known, a simple calculation using kinematics gives the distance traveled. These relationships can then be put into an excel spreadsheet, allowing us to quickly calculate the theoretical distance our ball will travel given a wide range of different masses (see appendix 3 for calculations)

Experimental results, Comparison, Discussion

The above calculations were applied and allowed us to come up with a set of theoretical data that we could analyze and compare experimental data to. This theoretical outcome is represented graphically below as a curve of expected distance traveled versus weight applied to the catapult. We conducted 10 experimental shots. 5 of these 10 shots were conducted using 30 lbs as our launching mass and the other 5 employed 40 lbs. Since the total travel distance for each launch varied, we took the average of the 5 shots for each weight and used that for comparison against theoretical values.
Theoretical Results | Experimental Results
---|---
Weight (lbs) | Distance Traveled (ft) | Weight (lbs) | Average of 5 trials Distance Traveled (ft)
15 | 9.9 | - | -
20 | 16.1 | - | -
25 | 20.7 | - | -
30 | 24.4 | 30 | 23.5
35 | 27.3 | - | -
40 | 29.8 | 40 | 28.7

For a 30-lb weight, our calculations suggested that we should expect the projectile to travel about 24.4 feet. Experimental data showed our projectile traveling 23.5 feet (on average). Our calculations did not account for slight gusts of wind, which were present on the day of experimentation. Based upon weather data acquired from Easterwood Airport at the time of launch, the winds were measured at 4 knots (about 4.6 miles per hour) with gusts up to 7 knots (8.05 mph) bearing roughly 170 degrees on a magnetic aviation compass. Our tests were conducted on Robelmont Drive in College Station, only a few miles away from Easterwood Field. Robelmont is oriented about 40-50 degrees off of the wind direction, thus taking the cosine of this wind magnitude we can find the relative velocity of the wind in relation to our projectile’s direction of motion. A wind gust of 5 knots could easily have played a role in the error we recorded between theoretical calculations and experimental data (please see concluding statements for full list of possible errors). When using the 40-lb weights, theoretical calculations predicted a traversed distance of 29.8 feet, while our actual measurements read 28.7 feet. Our error between theoretical and experimental calculations ranged between .6 ft and 1 ft error (see appendix 3 for calculations). Plotting the theoretical distance traveled versus weight applied curve, we can add our experimental data to the overall graphical interpretation – clearly, our data and theory show a strong correlation. Although our theoretical expressions neglected such outside forces as air resistance and wind gusts, we were successfully able to model the motion of the trebuchet and the resulting effect it had on a projectile using kinetics and kinematics.

Conclusion

The principles of dynamics can be used to analyze motion for many different situations and applications. The trebuchet and its projectile are just two small-scale applications of dynamics in our world. The trebuchet in this case was carefully designed to produce a somewhat simplistic, 2-dimensional motion about its pivot point. Using what we learned of dynamics in class, we were able to analyze many aspects of this 2-dimensional motion including the moment imparted by the use of various masses about the pivot point, angular momentum,
velocity, and acceleration. These aspects of motion could then be applied to the projectile in question, assuming that the throwing arm itself is an ideal rod and transmits energy and force without bending or any other loss. We also assumed that the trebuchet would create only motion in the (x) and (y) directions to somewhat simplify our calculations. Air resistance was assumed negligible due to the small size and relatively heavy weight of our baseball—it is designed to penetrate the air well. The fact that the ball would only stay in the air for a few seconds and would only travel 20-30 feet added credence to our assumption of zero air resistance. Upon actually conducting our calculations and testing the real-world trebuchet performance, we were able to predict very nearly where our projectile would land. Upon switching the amount of mass used to propel our projectile (effectively increasing the moment about the pivot point and exerting a greater force on the projectile), we were able to modify our earlier calculations to account for the greater moment and once again very nearly predict our projectile final impact location. We did experience a few errors in our real-world measurements. We assumed that the pivot point of the throwing arm on the trebuchet was frictionless, while in reality, even though we used grease in the bearing, there is still a small but noticeable amount of friction remaining. With each subsequent increase in mass used, the friction seemed to play a larger role and thus would account for some of the discrepancies between our theoretical data and that gathered through experiment. Sudden gusts of wind also played a role in our overall error. We picked relatively wind-free days for our test shots, but sudden gusts of wind while the ball is in the air would still play a very small role in our calculations. Last but not least, air resistance in the real world is most definitely not zero as we had assumed in our calculations. Although the ball would stay up in the air for a very short amount of time and travel a short distance, air resistance still has an effect on its overall travel. Overall, based upon the extremely close correlation between our theoretical calculations and real world analysis of data, this experiment was conducted very successfully.
Works Cited

Appendix A
\[ \frac{T}{I} = \alpha \]

\[ T = 2 F \cos \theta \hat{r} \]
\[ T = -4 F \cos \theta \hat{r} \]
\[ T = -6 F \cos \theta \hat{r} \]
\[ W_b = 4 F \cos \theta \hat{r} \]
\[ e = 1.2 \text{ in} \]
\[ W_b = 5.12 \text{ lb} \]
\[ w_B = 0.3175 \]
\[ g = 9.81 \text{ m/s}^2 \]

\[ N_B = -\cos \theta \ W_b \hat{e}_o \]
\[ T_{Bz} = \cos \theta \ W_b \ r_3 \ \hat{a} \]
\[ N_b = -\cos \theta \ W_b \hat{e}_o \]
\[ T_{Bz} = \cos \theta \ W_b \ r_2 \ \hat{a} \]
\[ N_w = -\cos \theta \ W_w \hat{e}_o \]
\[ T_{wz} = -\cos \theta \ W_w \ r_1 \ \hat{a} \]

\[ \frac{\Sigma T_z}{I} = \frac{(w_B r_3 + w_b r_2 - w_w r_1) \cos \theta \ \hat{a}}{r_3 \frac{w_B}{2} + r_2 \frac{w_b}{2} + r_1 \frac{w_w}{2}} = \alpha \]

\[ \alpha \text{ average} = 7.44 \text{ rad} \text{ per rad} \text{ per time} \]
\[ \alpha = \frac{\theta}{t} \]
\[ V_0 = \sqrt{2} a \left( \frac{\pi}{3} r \right) \]

\[ y = -g t^2 + \frac{\pi}{3} \cos \theta \ V_0 t + 8 F \]
\[ x = \cos \theta \ V_0 t + \]

\[ g = 9.81 \text{ m/s}^2 \]
\[ F = 1 \text{ N} \]
Trebuchet FBD's

Arm

$F_x$  $F_y$  pivot point  $w_{wood}$

$w_{projectile}$

base

$F_y$  $f_{friction}$  $N$

projectile

on arm

$N_{projectile}$  $f_{friction}$  $w_{projectile}$

in flight

$w_{projectile}$