AERO 214
Mohr’s Circle Procedure

1. Three possible starting points (given):
   a. stress cube given
   b. stress tensor
   c. determine stress state using conservation laws, etc.

2. Sign convention
   a. Stress cube – review (real world)
      1) Positive sign convention
         \[ \pm = \pm \text{ direction on } \pm \text{ face} \]
      2) How to indicate negative shear, e.g. \( T_{xy} = -2 \)
         \[
         \begin{align*}
         T_{xy} &= -2 \\
         \end{align*}
         \]
   b. In Mohr Space (not real world)
      Tension =+ for normals, CW couple =+ for shears

3. Basic steps
   a. Determine stress state if not given
   b. Draw stress cube if not given
   c. Calculate useful quantities, * depends on which plane of “plane stress”
      \[
      \begin{align*}
      \text{center} &= \frac{T_{xx} + T_{xy}}{2} \\
      \text{radius} &= \sqrt{\left(\frac{T_{xx} - T_{yy}}{2}\right)^2 + T_{xy}^2} \\
      \sigma_{p1} &= \text{center} + \text{radius} \\
      \sigma_{p2} &= \text{center} - \text{radius} \\
      \sigma_{p3} &= T_{zz} \\
      \end{align*}
      \]
      \[
      \begin{align*}
      \sigma_{p1} &\geq \sigma_{p2} \geq \sigma_{p3} \\
      \tau_{xy} &= \text{radius} \equiv \text{max. shear stress in plane}
      \end{align*}
      \]
d. Plot points,
   x-face \((T_{xx}, T_{xy})\)  \(\text{Note: one } T_{xy} = + \iff \text{sign convention } cw = +\)
   y-face \((T_{yy}, -T_{xy})\)  \(\text{one } T_{xy} = - \iff \text{ccw = -}\)
   z-face \((T_{zz}, 0)\)

e. Construct circle(s)

f. Determine \(\tau_{s_{\text{max}}}, \theta_p, \theta_s\) using trig.
   \[
   \tan \theta = \frac{\text{opp}}{\text{adj}}, \quad \sin \theta = \frac{\text{opp}}{\text{hyp}}, \quad \cos \theta = \frac{\text{adj}}{\text{hyp}}, \quad \tau_{s_{\text{max}}} \equiv \text{overall maximum shear stress}
   \]

   \(\theta_p \equiv \text{angle of rotation from original view to principal planes}\)

   \(45^\circ - \theta_p = \theta_s \equiv \text{angle of rotation from original view to in-plane maximum shear planes}\)

   \(\text{(i.e. where } \tau_{s_{\text{max}}} \text{ acts)}\)

   \[45^\circ - \theta_p = \theta_s \equiv \text{angle of rotation from original view to in-plane maximum shear planes}\]

   \(\text{(i.e. where } \tau_{s_{\text{max}}} \text{ acts)}\)

   \[45^\circ - \theta_p = \theta_s \equiv \text{angle of rotation from original view to in-plane maximum shear planes}\]

   \(\text{(i.e. where } \tau_{s_{\text{max}}} \text{ acts)}\)

g. Draw sketches of: principal planes & maximum shear planes

4. Comments on # of circles, etc.

   a. if \(T_{xx} = T_{yy} \neq T_{zz}\) and no shear stress, will generate just one circle (two circles on top of each other and a point)

   b. if \(T_{xx} = T_{yy} = T_{zz}\) and no shear stress, will degenerate to just a point

   c. for all other cases, no guarantees that “original” circle will be inside or outside of largest circle