Prob. 3.3-8. An axial load $P$ is applied to a tapered rod, as shown in Fig. P3.3-8. The radius of the rod is given by

$$r(x) = \frac{r_0}{1 + (x/L)}$$

where $r_0 = r(0)$. (a) Determine a symbolic expression for the elongation of this tapered bar in terms of the parameters $P$, $L$, $r_0$, and $E$, and (b) solve for the elongation in inches if $P = 2$ kips, $L = 100$ in., $r_0 = 20$ in., and the rod is made of an aluminum alloy for which $E = 10 \times 10^6$ ksi.

**Solution:**

(a) Expression for elongation of tapered rod.

From Eq. (3.11),

$$C = \frac{\int_0^L \frac{F(x)}{A(x)} \, dx}{E}$$

From the FBD,

$$-\sum F_x = 0 \quad P - F(x) = 0$$

$$F(x) = P = \text{const}$$

$$C = \frac{P}{E} \int_0^L \frac{dx}{\pi r(x)}$$

$$= \frac{P}{\pi E} \int_0^L \frac{dx}{\frac{r_0}{1 + (x/L)}^2} = \frac{P}{\pi E r_0^2} \int_0^L \left[ 1 + (x/L) \right]^2 \, dx$$

$$= \frac{P}{\pi E r_0^2} \left[ \int_0^L 1 + 2(x/L) + \frac{1}{3}(x/L)^2 \right] \, dx$$

$$= \frac{PL}{\pi E r_0^2} \left[ (xL) + \frac{1}{3}(xL)^3 \right]_0^L = \frac{PL}{3 \pi E r_0^2}$$

$$C = \frac{7 \pi E}{3 \pi E r_0^2} \quad \text{Ans. (a)}$$

(b) Elongation for specific case.

$$C = \frac{7 \times 2 \text{ kips}}{3 \pi \times (100 \text{ in.}) \times (10^6 \text{ ksi})} = \frac{37.136}{(10^{-4}) \text{ in.}}$$

$$C = 37.1 \times 10^4 \text{ in.} \quad \text{Ans. (b)}$$
**Prob. 3.3-9.** The tapered solid stone pier in Fig. P3.3-9 is 20 ft high, and it has a square cross section with side dimension that varies linearly from 36 in. at the top to 48 in. at the bottom. Assume that the stone is linearly elastic with modulus of elasticity \( E = 4.0 \times 10^3 \) ksi. Determine the shortening of the pier under a compressive load of \( P = 150 \) kips. (Neglect the weight of the stone.)

**Solution:**

Determine the shortening of the pier.

\[ \Delta = \frac{E}{A} \frac{dE}{dx} \]

From Eq. (3.11),

\[ \Delta = - \int_0^L \frac{F(x)}{A(x)} \, dx \]

From the FBD, \( F(x) = -P = \text{const} \)

**Geometry:**

From similar triangles,

\[ \frac{c-a}{L} = \frac{b(x) - a}{x} \]

\[ b(x) = \left[a + \left(c-a\right)\left(\frac{x}{L}\right)\right] \]

\[ A(x) = b(x)^2 \]

\[ \Delta = \frac{P}{E} \int_0^L \frac{dx}{[a + \left(c-a\right)\left(\frac{x}{L}\right)]^2} \]

\[ \Delta = - \frac{PL}{E(c-a)} \left[ \frac{1}{a + \left(c-a\right)\frac{x}{L}} \right]_0^L = - \frac{PL}{E(c-a)} \left[ \frac{1}{c} - \frac{1}{a} \right] \]

\[ \Delta = \frac{PL}{Eac} = \frac{(150 \text{ kips}) \cdot (240 \text{ in.})}{(4 \times 10^3 \text{ ksi}) \cdot (36 \text{ in.}) \cdot (48 \text{ in.})} = 5.2083 \times 10^{-3} \text{ in.} \]

\[ \Delta = 5.21 \times 10^{-3} \text{ in.} \quad \text{Ans.} \]
**Prob. 3.3-11.** A uniform circular cylinder of diameter $d$ and length $L$ is made of material with modulus of elasticity $E$ and specific weight $\gamma$. It hangs from a rigid "ceiling" at $A$, as shown in Fig. P3.3-11. Using Eq. 3.12, determine an expression for the (downward) vertical displacement, $u(x)$, of the cross section at distance $x$ from end $A$.

**Solution:**

\[ FBD: \text{Cylinder Segment} \]

\[ \Sigma F_x = 0: -F(x) + W(x) = 0 \]

\[ F(x) = \gamma A (L - x) \]

from Eq. 3.12,

\[ u(x) = u(x) + \int_0^x \frac{F(\xi)}{AE} \, d\xi \]

\[ = \frac{\gamma}{E} \int_0^x (L - \xi) \, d\xi \]

\[ = \frac{\gamma}{E} \left( Lx - \frac{x^2}{2} \right) \]

**Ans.**

\[ u(x) = \frac{\gamma}{E} \left( Lx - \frac{x^2}{2} \right) \]
Prob. 3.3-12. Nonuniform distributed axial loading of the circular rod $AB$ in Fig. P3.3-12 causes an extensional strain that can be expressed in the form $\varepsilon(x) = c_1 \left[ 1 - \left( \frac{x}{L} \right)^2 \right]$, where $c_1$ is a constant. (a) Determine an expression for the total elongation of the rod $AB$. (b) If the maximum axial stress in this rod is $\sigma_{\text{max}} = 15.0$ ksi, and its modulus of elasticity is $E = 10 \times 10^3$ ksi, what is the value of the constant $c_1$? (Give the proper units of $c_1$.) (c) Using your previous results, calculate the total elongation of rod $AB$ if its length is $L = 10$ ft.

**Solution:**

(a) Total elongation.

From Eq. 3.3,

$$\varepsilon = \int_0^L \varepsilon(x) \, dx = c_1 \int_0^L \left[ 1 - \left( \frac{x}{L} \right)^2 \right] \, dx = c_1 \left[ L - \frac{1}{L^3} \left( \frac{L^2}{3} \right) \right] = \frac{2c_1L}{3}$$

$$\varepsilon = \frac{2c_1L}{3} \quad \text{(Ans. (a))}$$

(b) Value of constant $c_1$.

$\varepsilon_{\text{max}}$ occurs at $x = 0$

$$\sigma_{\text{max}} = E \varepsilon_{\text{max}} = E c_1 \Rightarrow c_1 = \frac{\sigma_{\text{max}}}{E} = \frac{15.0 \text{ ksi}}{(10^4 \text{ ksi})} = 1.5 \times 10^{-3}$$

$$c_1 = 1.5 \times 10^{-3} \quad \text{(Ans. (b))}$$

Like strain $\varepsilon$, $c_1$ is a dimensionless quantity.

(c) Total elongation of 10 ft-long rod.

$$\varepsilon = \frac{2c_1L}{3} = \frac{2}{3} (1.5 \times 10^{-3}) (120 \text{ in.}) = 0.120 \text{ in.}$$

$$\varepsilon = 0.12 \text{ in.} \quad \text{(Ans. (c))}$$
Prob. 3.3-15. The stiffeners in airplane wings may be analyzed as uniform rods subjected to distributed loading. The stiffener shown below has a cross-sectional area of 0.60 in$^2$ and is made of aluminum ($E_s = 10 \times 10^6$ psi). Determine the elongation of section $AB$ of the stiffener, whose original length is 20 in., if the stress in the stiffener at end $A$ is $\sigma_A = 5,000$ psi, and a uniform distributed loading of 40 lb/in. is applied on either side of the stiffener over the 20-in. length from $A$ to $B$.

Solution:

FBD: Stiffener segment

Equilibrium

\[ F(x) = (0.60 \text{ in}^2) \sigma_A + (40 \text{ lb/in.}) x \]

\[ = 3000 \text{ lb} + (40 \text{ lb/in.}) x \]

\[ e = \int_0^L \frac{F(x) \, dx}{AE_s} \]

\[ = \frac{1}{AE_s} \int_0^L (3000 + 40 x^2) \, dx \]

\[ = \frac{1}{AE_s} \left[ 3000 x + 40 \frac{x^3}{3} \right]_0^L \]

\[ = 0.012667 \text{ in.} \text{ Ans.} \]
**Prob. 3.3-16.** The trapezoidal flat bar in Fig. P3.3-16 is suspended from above and loaded only by its own weight. The thickness of the bar is constant, and its width varies linearly from width $b_0$ at the bottom to $2b_0$ at the top. The weight density of the material is $\gamma$. For the following case, calculate the normal stress on the cross section midway between the bottom and top of the bar: $b_0 = 10$ in., $L = 20$ ft, and $\gamma = 0.284$ lb/in$^2$.

**Solution:**

Determine the axial stress midway between the top and bottom of the bar.

**Geometry:**

Width $W$: $b(L/2) = \frac{1}{2}(b_0 + \frac{3}{2}b_0) = \frac{5}{2}b_0$

Area: $A(L/2) = \frac{3}{8}b_0 t$

Volume: $V(L/2) = \frac{1}{3} \left[ \frac{1}{2}(b_0 + \frac{3}{2}b_0) \right] t = \frac{5}{8}b_0 t L$

**Equilibrium:**

\[ \sum F = 0 \]

\[ F(L/2) = W(L/2) = \gamma V(L/2) = \frac{5}{8}b_0 t L \]

**Axial stress:**

\[ \sigma(L/2) = \frac{F(L/2)}{A(L/2)} = \frac{5}{8} \left( \frac{b_0 t L}{\frac{3}{8} t} \right) = \frac{5}{12} \gamma L \]

\[ \sigma(L/2) = \frac{5}{12} \gamma L \]

\[ \sigma(L/2) = 28.4 \text{ psi} \quad \text{Ans.} \]